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The Mechanism and Structure of Planetary Nebulæ.

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1. A quite cursory glance at a series of photographs of planetary nebulæ suffices to show that their forms cannot be explained as those of masses of gas in equilibrium under gravitation and rotation. The suggestion has often been made that light-pressure may play a part in the mechanism of these nebulæ, and the object of the present note is to examine to what extent nebular forms can be explained as the result of the joint action of gravitation, rotation, and light-pressure resulting from the radiation of a central star. As regards positive results the investigation is perhaps disappointing, at least in comparison with what I had at first hoped, but I have thought it worth publishing as an explanation of hitherto unexplored territory.

2. Let the central star be of mass M and of radius a, and let it radiate energy at a rate E per unit time. Then the intensity of radiation at its surface, measured in ergs per sq. cm. per second, is

$$\mathrm{I} = \frac{\mathrm{E}}{4\pi a^2}.$$

If there were no absorbing matter present, the intensity would fall off as the inverse square of the distance. Let us, however, suppose that there is absorbing matter arranged in spherical layers around the central star, the matter in any shell of radius r being of density ρ and opacity κ. Then it is readily shown that I must satisfy the differential equation

$$\frac{\partial \mathrm{I}}{\partial r} = -\mathrm{I}\left(\frac{2}{r} + \kappa\rho\right),$$

whence

$$\mathrm{I} = \frac{\mathrm{EA}}{4\pi r^2}$$

where

$$\mathrm{A} = e^{-\int_a^r \kappa\rho\, dr} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Clearly (1 – A) measures the fraction of light which is absorbed by traversing the absorbing matter between spheres of radii a and r.

Consider any small area $r^2 d\omega$ of the shell of radius r. A thickness dr of this area absorbs momentum from the radiant energy at a rate

$$\frac{I}{C}\kappa\rho\, dr\, r^2\, d\omega \text{ per second,}$$

so that the equation of motion of the element, in the direction radially away from the central star, is

$$\frac{d^2r}{dt^2}\rho\, r^2\, dr\, d\omega = -\frac{\partial p}{\partial r}r^2\, dr\, d\omega - \frac{\gamma M}{r^2}\rho\, r^2\, dr\, d\omega + \frac{I}{C}\kappa\rho\, dr\, r^2\, d\omega\,;$$

or, dividing throughout by $\rho r^2\, dr\, d\omega$, and substituting the value just found for I,

$$\frac{d^2r}{dt^2} = -\frac{1}{\rho}\frac{\partial p}{\partial r} - \frac{\gamma M}{r^2} + \frac{\kappa EA}{4\pi r^2 C} \quad . \quad . \quad . \quad . \quad (2)$$

3. Let us now examine whether, in the absence of rotation, it is possible for the absorbing matter to stand in equilibrium in spherical layers surrounding the central star. The equation of equilibrium for such a state, if possible, is obtained by equating the right-hand member of equation (2) to zero, and so is

$$\frac{\partial p}{\partial r} = -\frac{\rho}{r^2}\left[\gamma M - \frac{\kappa EA}{4\pi C}\right] \quad . \quad . \quad . \quad . \quad (3)$$

The value of p at the surface of the star, as also at $r = \infty$, is $p = 0$. Thus if it is possible for detached shells of matter to stand in equilibrium surrounding the star, p must rise to a maximum value somewhere between $r = a$ and $r = \infty$. The value of r, if any, for which p attains this maximum value is given by $\partial p/\partial r = 0$, or

$$4\pi C\gamma = \frac{E}{M}\kappa A \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Since A is necessarily less than unity, it is clear that p cannot have a maximum at all unless

$$\frac{E}{M} > \frac{4\pi\gamma C}{\kappa} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

If this inequality is not satisfied p cannot have a maximum value at all, and so cannot rise above zero at any point outside the surface $r = a$: thus the structure is merely an ordinary star. If this equality is satisfied, a shell of matter, entirely detached from the star, can rest in equilibrium as a sphere surrounding the star. According to the hypothesis we are now examining, the simplest type of planetary nebulæ is formed of a central star with such an enveloping sphere of matter.

4. The value of $4\pi\gamma C$ is about 25,000; for our sun E/M is about 2. Thus our sun cannot hold a detached shell of matter in suspension unless κ is greater than about 12,500. For a giant star E/M may be 100 times as great as for our sun, so that here the minimum value of κ would be

125. These values may be compared with Eddington's* estimate of $\kappa = 100$ as the maximum admissible value of κ for ordinary stellar matter. It must, however, be remembered that Eddington's estimate refers to matter at a temperature of about 1,000,000° C. traversed by radiation of wave-length corresponding to this temperature; the physical conditions in the planetary nebulæ are necessarily widely different.

5. It is extraordinarily difficult to form any estimate of the value of E/M for the central stars of the planetary nebulæ. Van Maanen,† having measured the parallaxes of six planetary nebulæ, calculates the absolute magnitudes of the central stars as $+6{\cdot}7, +9{\cdot}2, +10{\cdot}1, +10{\cdot}4, +8{\cdot}8, +9{\cdot}7$, the mean of the six being $+9{\cdot}1$, with comparatively small dispersion. Thus he finds the value of E to be only about one-fortieth of its value for our sun. Most of the planetary nebulæ show a pronouncedly spheroidal form, and according to the spectroscopic determinations of Campbell and Moore ‡ these nebulæ are, in every case in which a positive result can be obtained, in rotation about the minor axis of the spheroid. Assuming that their spheroidal form results from a balance between gravitation and rotation, a combination of the spectroscopic velocities of rotation found by Campbell and Moore with the linear dimensions calculated (from his parallax determinations) by Van Maanen leads to values for the mass of the central star varying from five to fifty times that of the sun.§

It is difficult to reconcile the low values obtained for the luminosity with the high values obtained for the mass. The difficulty is increased beyond measure by the circumstance that the spectra of the central stars are of types O and B. No readjustment of the assumed parallaxes will remove the difficulty: to obtain luminosities of the order of those of the average B-type star, we should have to suppose that Van Maanen's parallaxes were too large by a factor of about 100, while to obtain masses of the order of those of the average B-type star, we should have to suppose that the same parallaxes were too small by a factor of about 100, the adjusted parallaxes having the quite impossible values of about two seconds of arc.

6. Abandoning this unprofitable line of attack, we may perhaps approach the problem anew from the observed fact that the light emitted by the central stars of the planetary nebulæ contains an abnormally large proportion of violet light. Thus W. H. Wright,‖ discussing the spectra of the nuclei of the planetary nebulæ, says: "The most striking feature of the continuous spectrum is its very uniform extension into the ultra-violet. I have observed no exception to this rule. While there may be slight variations in the character of the intensity curves, speaking generally, any one of them may be taken as a type of all." He further gives a typical spectrum corrected approximately for the absorption of the atmosphere, and it is found that the intensity increases steadily as

* *M.N.*, **77** (1917), 600.

† *Proc. Nat. Acad. of Sciences*, **4** (1918), 394 (Mount Wilson Contribution, No. 56).

‡ *Proc. Astron. Soc. Pacific*, **27** (1915), and **28** (1916); and *Lick Obs. Publications*, **13** (1918), 77.

§ Campbell and Moore, *Lick Obs. Publications*, **13** (1918), 177.

‖ *Lick Obs. Publications*, **13** (1918), 251.

the frequency diminishes, being twice as great at $\lambda 3400$ as at $\lambda 4000$. Even this probably underestimates the preponderance of ultra-violet light, since no account is taken of the diminished sensitiveness to ultra-violet light of the plates (S 23) used in his observations. He concludes: "where the maximum lies cannot be conjectured."

Now the estimate of E given in the last section had reference only to photographic light of sufficiently low frequency to penetrate our atmosphere, and the true value of E may be widely different. But, whatever the value of E, it seems quite possible that the frequency of the light mainly emitted may be in the neighbourhood of strong absorption frequencies for particular substances. For these substances and for this light κ may be very large indeed, and there seems to be no objection to assuming that equation (5) may be satisfied. We must, however, from now on assume that E is not the total emission of radiation, but is the emission of radiation within a certain definite range of high frequency, and that κ is the opacity of matter of a special kind for light of this frequency.

7. In the special case in which there is no rotation and the total amount of matter suspended in equilibrium is small, this matter will form a thin spherical shell surrounding the sphere (say) $r=b$. If we replace A by its value (as given by equation (1)), and integrate equation (3) through the whole thickness of the shell, from $p=0$ on one side to $p=0$ on the other side, assuming the shell to be so thin that r does not differ appreciably from b throughout the range of integration, we obtain

$$0 = -\frac{\gamma M}{b^2}\int \rho dr + \frac{E}{4\pi b^2 C}\int e^{-\int_a^r \kappa\rho\, dr}\kappa\rho\, dr$$

$$= -\frac{\gamma M}{b^2}\int \rho dr + \frac{E(1-A_1)}{4\pi b^2 C} \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where A_1 is the value of A at the outer surface of the shell. The total mass of the suspended shell is given by

$$m = \int 4\pi\rho r^2 dr = 4\pi b^2 \int \rho dr = \frac{E b^2 (1-A_1)}{\gamma MC} \quad . \quad . \quad (7)$$

For the six planetary nebulæ already mentioned, Van Maanen finds an average diameter ($2b$) of about 4000 units, corresponding to $b = 3\times 10^{16}$ cms. The value of γC is about 2000, while $1-A_1$ may, with sufficient accuracy for our present purpose, be taken equal to $\frac{1}{2}$. Substituting these values, equation (7) gives

$$m = 2{\cdot}2\times 10^{29}\times\frac{E}{M}\text{ grammes.}$$

In the absence of precise knowledge as to the value to be assigned to E/M we cannot make any definite estimate as to the value of m, but it is clear that this is of stellar order of magnitude.

8. Let κ_0 denote the average value of κ throughout the shell, and let us write

$$x = \int \kappa\rho\, dr = \kappa_0 \int \rho dr = \frac{\kappa_0 m}{4\pi b^2} \quad . \quad . \quad . \quad . \quad (8)$$

so that $A_1 = e^{-x}$. Then equation (7) assumes the form

$$x = \frac{\kappa_0 E}{4\pi\gamma MC}(1 - e^{-x}) \quad . \quad . \quad . \quad . \quad (9)$$

When E/M is known this equation determines x, and so gives m if b is known, or b if m is known. Regarded as an equation in x, it has no root at all if $\kappa E/4\pi\gamma MC$ is less than unity, bringing us back to inequality (5) as the condition that free suspension can occur at all. When $\kappa E/4\pi\gamma MC$ is equal to unity the equation has the single root $x = 0$; when this quantity is greater than unity there is one and only one positive root for x, the value of which ultimately becomes identical with that of $\kappa E/4\pi\gamma MC$ when this latter quantity is very large.

Thus when m is fixed an increase in E results in a decrease in b, and *vice versa*. As the evolution of the central star progresses E may be expected to decrease, so that the surrounding sphere of matter will be driven further and further away until finally, when E attains the critical value $4\pi\gamma MC/\kappa$, it passes out of the star's influence altogether. Passing backwards through the star's past history we must ultimately come to a time at which the radius b of the shell coincided with the radius a of the star. Previous to this the shell must have formed an integral part of the star.

In actual fact m is not likely to remain fixed, on account of the escape of atmosphere from the outer surface of the shell. The smallest value permissible for M, if we accept Campbell's radial velocities and Van Maanen's distances already mentioned, appears to be of the order of from 10^{34} to 10^{35}. Taking $M = 10^{35}$ and $b = 3 \times 10^{16}$, the velocity of escape is found to be about 6·6 kms. a second. With $M = 10^{34}$ the velocity of escape would be 2·1 kms. a second. These values do not suggest rapid escape, but there must be some, and as it occurs we may notice that x will remain constant, so that b^2 will be proportional to m. It seems possible that the radii of the shells of planetaries may, in part at least, be determined by the condition that the escape shall be slight, so that whenever the decrease of E would lead to a value of b at which escape would be large, the initial stages of this escape reduce the value of m and so check the tendency for b to increase. This hypothesis seems to give some approach to an explanation of why the radii of planetaries are what they are observed to be rather than something else.

9. Still assuming the shell to be thin and spherical, let us examine the stability of its suspension. If we integrate the general equation (2) throughout the thickness of the shell, we obtain in place of equation (6)

$$\frac{d^2r}{dt^2} = -\frac{\gamma M}{r^2} + \frac{E}{4\pi r^2 C}\frac{1 - A_0}{\int \rho dr}.$$

If $r = b$ is the equilibrium position of the shell, the equation of motion of any element of the shell when at $r = b + \delta r$ is

$$\frac{d^2}{dt^2}(\delta r) = \frac{2\delta r}{b}\left[\frac{\gamma M}{r^2} + \frac{E}{4\pi r^2 C}\frac{1 - A_0}{\int \rho dr}\right]_{r=b} + \frac{E\delta r}{4\pi b^2 C}\frac{\partial}{\partial r}\left\{\frac{1 - A_0}{\int \rho dr}\right\}.$$

The term in square brackets vanishes as the condition that $r = b$ is a position of equilibrium. If we return to the notation of equations (8), our equation reduces to

$$\frac{d^2}{dt^2}(\delta r) = \frac{\mathrm{E}}{mb^2\mathrm{C}}\delta r\frac{\partial}{\partial r}\{r^2(1-e^{-x})\}$$
$$= \frac{\mathrm{E}}{m\mathrm{C}}\delta r\left\{\frac{2}{b}\left[1-e^{-x}-xe^{-x}\right]+xe^{-x}\frac{1}{\kappa_0}\frac{d\kappa_0}{dr}\right\} \quad . \quad . \quad (10)$$

The term in square brackets is positive for all values of x, so that if κ does not vary with r the suspended shell will be unstable for all radial displacements. In general, however, the value of κ_0 will depend upon the physical condition of the absorbing matter, and in particular upon its temperature. As r decreases the temperature will increase, and if κ increases sufficiently rapidly with the temperature, $d\kappa_0/dr$, which will be negative, may be sufficiently large numerically to determine the sign of the right-hand member of equation (10). In this case the equilibrium will be stable.

When either m is small or b is large, x is a small quantity and equation (10) reduces to

$$\frac{d^2}{dr^2}(\delta r) = \frac{\mathrm{E}}{4\pi b^2\mathrm{C}}\frac{d\kappa_0}{dr}(\delta r).$$

In this case even the smallest negative value of $d\kappa_0/dr$ is sufficient to ensure stability.

10. We next consider the conditions for relative equilibrium of a shell rotating with a steady uniform angular velocity ω. Taking polar co-ordinates r, θ, ϕ with $\theta = 0$ for axis of rotation, the components of acceleration of an element at r, θ, ϕ are

$$-\omega^2 r\sin^2\theta, \quad -\omega^2 r\sin\theta\cos\theta, \quad 0 \quad . \quad . \quad . \quad (11)$$

whence, by equation (2), the equations of relative equilibrium are

$$\frac{1}{\rho}\frac{\partial p}{\partial r} = \omega^2 r\sin^2\theta - \frac{\gamma\mathrm{M}}{r^2} + \frac{\kappa\mathrm{EA}}{4\pi r^2\mathrm{C}} \quad . \quad . \quad . \quad (12)$$

$$\frac{1}{\rho}\frac{\partial p}{\partial\theta} = \omega^2 r^2\sin\theta\cos\theta \quad . \quad . \quad . \quad . \quad . \quad (13)$$

At the inner surface of the suspended shell $\mathrm{A} = 1$. Thus if we put

$$\Phi = \tfrac{1}{2}\omega^2 r^2\sin^2\theta + \frac{\gamma\mathrm{M}}{r} - \frac{\kappa\mathrm{E}}{4\pi r\mathrm{C}} \quad . \quad . \quad . \quad (14)$$

it appears that at the inner surface equations (12) and (13) assume the forms

$$\frac{1}{\rho}\frac{\partial p}{\partial r} = \frac{\partial\Phi}{\partial r}; \quad \frac{1}{\rho}\frac{\partial p}{\partial\theta} = \frac{\partial\Phi}{\partial\theta},$$

so that the inner surface ($p = 0$) must be one of the family of surfaces

$$\Phi = \text{constant} \quad . \quad . \quad . \quad . \quad . \quad (15)$$

11. Let us consider mathematically the shapes of the family of surfaces

$$\tfrac{1}{2}\omega^2(x^2+y^2)+\frac{B}{r}=K \quad . \quad . \quad . \quad . \quad (16)$$

which are equivalent to the family $\Phi =$ constant, when ω^2, B, and K are constants. Although the equation contains three constants, its significance is not altered by increasing or decreasing these constants in the same ratio, so that the family of surfaces is only doubly infinite. Further, if

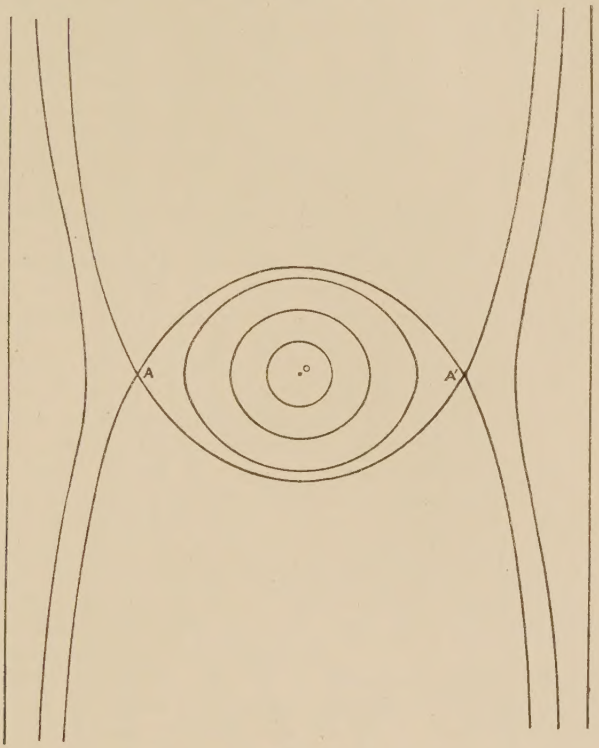

FIG. 1.

K is kept fixed, the whole family is obtained by giving all possible values to ω^2 and B, but variations in ω^2 and B such that the product ωB retains a constant value result only in changes in the size of the surface without any accompanying change of shape. Thus as regards their shape only, the surfaces will form a singly infinite family.

When B = 0 the equation reduces to $x^2+y^2 =$ cons., and the surface is a cylinder surrounding the axis of rotation. When B is positive the surfaces are identical with the well-known family, first discussed by Roche,* which gives the possible external boundaries for a mass of

* *Acad. de Montpellier* (*Sciences*), 8 (1873), 235. See also Jeans, *Problems of Cosmogony*, p. 147.

tenuous gas rotating under the influence of a gravitating mass B at the origin. These surfaces are represented in fig. 1, their limit inwards being a sphere surrounding the origin and their limit outwards being a cylinder corresponding to the value $B = 0$.

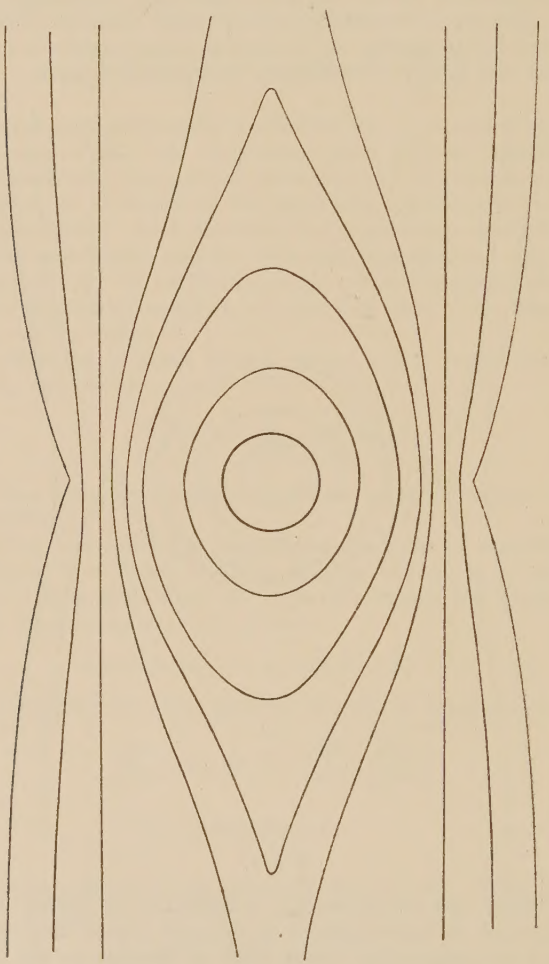

FIG. 2.

When B is negative the surfaces are represented in fig. 2. The limit inwards is again a sphere surrounding the origin, and the limit outwards is again the cylinder $B = 0$. The surfaces outside this cylinder in fig. 2 are merely the open surfaces reproduced from fig. 1, and these of course correspond to positive values of B.

12. Turning back to equation (14), it appears that any one of the

surfaces shown in figs. 1 and 2 may represent the inner surface of the nebular shell provided only that $\partial p/\partial r$ is negative, a condition which requires that K shall increase as we pass outwards. This condition merely rules out the closed surfaces in fig. 1, so that the possible boundaries for the inner surface are precisely the surfaces shown in fig. 2.

It is rather less easy to discuss the possible forms for the outer surface. For the sake of simplicity let us suppose that the only light whose mechanical action is appreciable is that of one particular colour, say ultra-violet, and let us suppose that this is practically all absorbed within the thickness of the shell. To adjust our analysis to this assumption we must suppose that E has reference to the emission of ultra-violet light only, and we take $A = 0$ at the outer surface of the shell. By an argument precisely similar to that of § 10 we find that the outer boundary may be any one of the surfaces $\Phi = \text{constant}$, when Φ is given by equation (14) with the term in E omitted. Thus for the outer surface, B, which is now equal simply to γM, is necessarily positive. This, together with the condition that $\partial p/\partial r$ must be negative, restricts the outer surface to being one of the closed surfaces shown in fig. 1.

A shell bounded by any of the surfaces shown in fig. 2, drawn on any scale whatever, as inner surface, and by any of the closed surfaces shown in fig. 1, drawn on any scale whatever, as outer surface, forms a possible figure of equilibrium. For the choice of the first surface restricts us merely to a certain ratio of ω^2 to $\gamma M - KE/4\pi C$, while the choice of the second surface restricts us to a certain ratio of ω^2 to γM. The double choice can accordingly always be justified by certain calculable values of ω^2, M, and E. The shells which are physically possible are of course only those for which ω^2, M, and E have positive values, but configurations which would give negative values to these quantities are seen to be physically impossible by a mere inspection. Examples of the various types of cross-section which can be obtained in this way are shown in fig. 3; the central star is shown black in each case; the vertical through this is the axis of rotation of the star and also the axis of revolution of the figure.

A general similarity can be at once recognised between these drawings and the forms of a certain number of well-known planetary nebulæ. Under each drawing will be found the N.G.C. numbers of nebulæ, which show at least a rough correspondence with the drawings, the numbers in brackets referring to the drawings of these nebulæ in Curtis' study of the planetary nebulæ.* But before any detailed comparison can be made, it is necessary to take full account of the effects produced by varieties of orientation in space and of various distributions of luminosity in the nebular matter.

13. The photographs of practically all nebulæ show an outer boundary of roughly elliptical shape, so that, except for the few nebulæ of approximately circular shape, we are justified in speaking of the major and minor axes of the photographic image of a nebulæ. From the extensive spectroscopic investigations of Campbell and Moore †

* *Lick Observatory Publications*, 13 (1918), 57.

† *Ast. Soc. Pacific*, vols. 27 (1915) and 28 (1916).

already referred to, it appears to be a general rule that when the slit of the spectroscope is set parallel to the minor axis no rotation is observed, whereas rotation is frequently observed when the slit is set parallel to the major axis. Thus the axis of rotation of a nebula and the minor

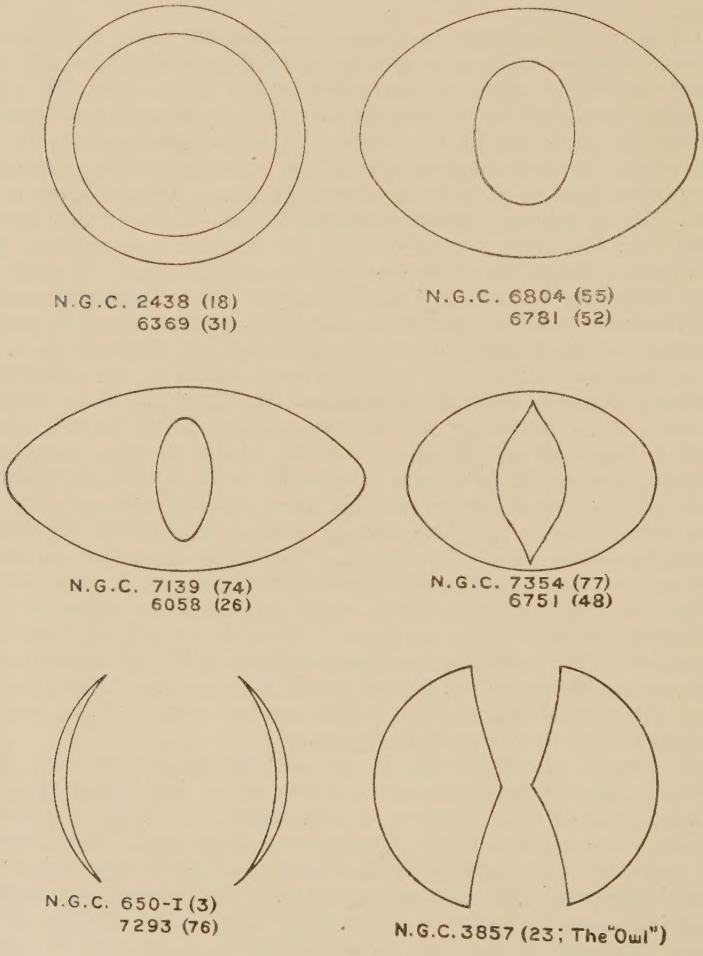

FIG. 3.

axis of its projection on the celestial sphere must lie in a plane passing through our eyes, or, to express the conclusion somewhat less accurately, the nebula is rotating about its minor axis.

In fig. 4 let the dotted line represent a section of the nebula by a plane containing the direction of sight EO and the minor axis POP′ of the photographic image of the nebula. Then the axis of rotation

ZOZ′ is also in this plane, and the line AOA′ which is perpendicular to ZOZ′ in this plane will be the section of the equator of the nebula by this plane.

The actual emission of light from the nebula in space may be expected to be symmetrical about the equator AOA′, but the intensity of light in the photographic image is found to be invariably symmetrical about the plane BOB′ through the line of sight. Now, if the nebulæ were opaque or partially opaque, the light from the half PAP′ of the nebula would affect the photographic plate more strongly than the light from the half PA′P′. Thus, except in the exceptional case in which AOA′ happened to lie along BOB′—*i.e.* the case in which we happen to lie in the equatorial plane of the nebula—the image of the nebula ought to show a want of symmetry about EBOB′. The circumstance that no such want of symmetry can be detected leads to the conclusion that the

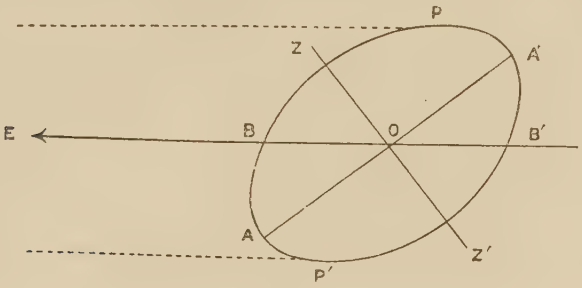

Fig. 4.

body of the nebulæ must be, for all practical purposes, completely transparent. Thus light from A′ must impress the photographic plate just as powerfully as the equal light from A, and so the observed symmetry is established. This interpretation of course requires us to suppose that the light which is active photographically and visually is quite distinct from that of which the light pressure is mechanically active. There is no objection to this; we have already conjectured that the light pressure involved may be that of ultra-violet light, and the interpretation just reached rather confirms this conjecture.

14. It being settled that the light received must be treated as a sum of contributions, each undimmed by absorption, from each part of the nebula in which matter is present, we may turn next to the relative amounts of the contributions made by the different elements of the nebula.

The simplest hypothesis it is possible to make is that equal volumes of matter emit equal amounts of light. In the case of a nebula formed by concentric spherical shells of radii a, b, this hypothesis requires that the intensity of light at a distance r from the centre should be (except for a constant multiplier)

$$\begin{aligned} &(a^2 - r^2)^{\frac{1}{2}} - (b^2 - r^2)^{\frac{1}{2}} \quad \text{when} \quad r < b, \\ &(a^2 - r^2)^{\frac{1}{2}} \qquad\qquad\quad \text{when} \quad r > b. \end{aligned}$$

There is a maximum brightness at $r = b$, and this is equal to $\sqrt{\left(\frac{a+b}{a-b}\right)}$ times the brightness at the centre $r = 0$.

Curtis has tested this prediction in the case of 38 nebulæ, and finds that in 25 out of the 38 the maximum brightness is such as can be accounted for by the observed values of a and b, while in 13 cases the hypothesis fails completely, the reason in every case being that the observed maximum brightness is too great to be accounted for by the observed values of a and b.

A slightly more plausible hypothesis is that equal masses of matter emit equal amounts of light. According to the view which has been put forward in the present investigation, the density of matter must vanish at both the inner and outer surfaces, and will attain a maximum in the interior of the matter of the shell. This hypothesis admits of considerably larger maximum brightnesses, and seems capable of accounting for a large proportion of the 13 cases in which Curtis has found the simpler hypothesis to fail.

Nevertheless, neither of these hypotheses can contain the whole truth. For, according to either of them, the observed luminosity ought to fall gradually to zero at the outer edge of the planetary. Nebulæ are known in which this gradual fading away occurs (*e.g.* N.G.C. 2610 (21), N.G.C. ii. 3568 (25)), but these cases form the exception rather than the rule. The normal event is for the brightness to rise to a maximum at the outer edge of the nebula (*e.g.* N.G.C. 3242 (22), N.G.C. 6804 (55)). A hypothesis which meets these cases is that the emission of light is confined to those regions in which the density is less than a certain critical density ρ_0. An analogy, although probably nothing more than an analogy, can possibly be found in the occurrence of auroral light in the earth's atmosphere; this seems to be quite sharply limited to regions in which the density is less than a certain critical density.

In certain nebulæ the whole shell may be of density less than ρ_0, in which case there will be an emission of light from the whole volume of the shell. In other nebulæ the density in the shell may rise to values many times the critical density, and in these the emission of light will be practically confined to two surfaces, namely, the inner and outer surfaces of the shell of suspended matter. Moreover, in the limiting case in which these regions may be treated as true surfaces, the image on a photographic plate will consist simply of two bright curves representing the cross-sections of the cylinders enveloping the inner and outer surfaces of the shell of suspended matter.

This hypothesis, which must of course be regarded only as an exact representation of the facts in very extreme cases, appears capable of explaining the curves observed in many planetary nebulæ. Further hypotheses are necessary to account for the variations in intensity in the emitted light, but these need hardly concern us here. We may just notice, however, that regions remote from the centre of the nebula will receive only faint light from the central star and so may be expected to emit light only faintly: this may account for the almost universal dimness observed at the ends of the major axis.

15. This concludes our theoretical discussion. It will be understood

that the object of the investigation has not been to advocate that any, or still less all, planetary nebulæ can be accounted for in terms of the action of light pressure, but rather to examine theoretically what the effects of this action are, in order that others, more skilled than myself, may compare them with observation. The agencies we have considered, namely, light pressure, gravitation, and uniform rotation, can of course only give rise to figures completely symmetrical about the axis of rotation. This, in combination with our supposition that the nebular matter is practically transparent to the light by which we see it, shows that our investigation can at most explain those nebulæ which appear on the photographic plate as symmetrical about both the major and minor axes. Apart from the simple shell and so-called "ring" nebulæ, the classes of nebulæ which the investigation would seem *prima facie* to be most capable of explaining are those typified by N.G.C. 7139 (74) and N.G.C. 7354 (77), to which perhaps may be added N.G.C. 3587 (23). The slightly wider hypothesis of two shells rotating with different velocities or of a single shell rotating with non-uniform rotation may possibly account further for nebulæ of the general type of N.G.C. 3242 (22) and N.G.C. 7009 (70). On the other hand, no widening of our fundamental hypothesis seems able to account for the large number of planetaries, such as N.G.C. 3242 (22), whose main visual feature consists of two surfaces both flattened out in the direction perpendicular to the axis of rotation. Our equation (13) shows that p must increase as we pass from the pole to the equator along any sphere $r = \text{constant}$; hence the inner surfaces of the shell, assuming one to exist, ought to show extension along the axis of rotation (*cf.* figs. 2 and 3). This seems to be the main difficulty confronting the hypothesis examined in the present paper.

The Development of Faint Images by Fogging. By S. R. Pike. (Plate 17.)

(*Communicated by Professor Lindemann, F.R.S.*)

It has long been known * that a faint photographic image can be brought out much more clearly by slightly fogging the plate, but up to the present the experiments which have been made on this subject have been mostly of a qualitative nature.

It is possible, however, by means of a photo-electric cell and an optical wedge, to measure densities on a numerical scale independently of any eye estimates; and it is the object of the present paper to describe this new and more accurate method of dealing with the problem, and to indicate its scope by giving the results obtained by its use in one particular case.

An arrangement was adopted by which a plate could be exposed in strips, parallel either to its length or to its breadth, for various periods

* Wood, *Astrophysical Journal*, 1908 June; Conrady, *The Observatory*, 1923 January.

of time, to a fixed source of light, so that there were six strips A B C D E F parallel to the breadth with spaces between them equal to their own breadth, and five strips 1 2 3 4 5 parallel to the length and similarly spaced. The exposure times were so chosen that the blackenings fell off progressively from A to F and from 1 to 5. The five strips 1 to 5 were taken to represent the faint images to be brought up, and the strips A to F the foggings employed to do so. The darker spots at the junctions of the strips gave the blackenings of the "brought up" images, and the portions of the A to F strips on either side of any one of the spots gave the fogging blackening that actually brought out this particular image.

The difference between the numerical values for each of these blackenings or densities gave then the "contrast" of the "brought up" image.

By this means each plate was made to give five separate curves, corresponding to images of five different densities, each curve being determined by six points on it.

In order to measure the blackenings, a constant current was passed through an electric lamp whose light was concentrated by a large lens and allowed to fall on a photo-electric cell after first passing through any chosen spot on the plate. The cell was connected up to a string galvanometer, which was read through a microscope with a travelling cross-wire. When the galvanometer readings for the required portions of the plate had been obtained, the plate was replaced by an Ilford optical wedge, and any given galvanometer reading could be reproduced by sliding the wedge along under the spot of light on a carrier, which was provided with a millimetre scale and vernier. By this means the absorptive power of any part of the plate was matched with that of a definite portion of the wedge.

By measuring the unexposed parts of the plate we can derive also the absolute blackenings of the fogging and image exposures. The numbers obtained in this way are more satisfactory for drawing curves from than the actual exposure times are; and further, as we shall see later, curves drawn in this way have a very definite importance of their own.

We define the density of an image as

$$D = \log_{10} \frac{\text{Intensity of incident light}}{\text{Intensity of transmitted light}}.$$

Then let D_c = density of unexposed parts of the plate.
D_i = density of unfogged image.
D_b = density of fogged image.
D_f = density of fogging image.

The quantities with which we are concerned are in each case difference of densities, or contrast. The apparent density of the unfogged image is $\delta_i = D_i - D_c = \log_{10} \frac{I_c}{I_i}$, where I_c is the intensity of the light transmitted by the unexposed parts of the plate, and I_i that transmitted by the unfogged image.

Now we know that these densities correspond to points on the wedge whose distance d_1 cms. apart has been measured; and consequently, since the wedge constant was 0·1 per cm., we have:

$$\log_{10} \frac{I_e}{I_i} = 0{\cdot}1 \times d_1.$$

Hence $\delta_i = 0{\cdot}1 \times d_1$, and similarly we find δ_b the apparent density of the fogging image, and finally $\Delta = D_f - D_b$, the contrast of the fogged image on the background of the fogging image, which is the quantity whose variations we are investigating. All these quantities are thus proportional to the differences of the scale numbers giving the various positions of the wedge.

In the investigation to be described attention has been confined strictly to a particular type of plate, viz. the Ilford panchromatic process plate. These are not particularly fast, but were chosen on account of their comparative freedom from chemical fog. Red light, to which they are very sensitive, was used, and the developer was Azol, generally diluted to one part in a hundred.

It was found that when a current of 1·0 amp. was passed through the lamp (12 volt ½ watt), and the latter was at a distance of about 20 feet from the plate, then the following exposures

A.	B.	C.	D.	E.	F.	1.	2.	3.	4.	5.
12^s	11^s	10^s	9^s	8^s	7^s	5^s	4^s	3^s	2^s	1^s

gave curves whose maxima were all included in the central part of the plate.

The actual figures were as follows:—

Density of Faint Image:

1.	2.	3.	4.	5.
0·54	0·23	0·13	0·085	0·01

These numbers, of course, represent the densities of the strips 1–5 at the points where they do *not* cross the A – F strips, but relative to the unexposed parts of the plate.

Density of Fogging Exposure:

A.	B.	C.	D.	E.	F.
2·4	2·19	1·94	1·64	1·34	0·99

These numbers represent the corresponding quantities for the A – F strips.

Contrast:

	A.	B.	C.	D.	E.	F.
1.	1·30	1·41	1·44	1·50	1·78	1·53
2.	1·01	1·15	1·10	1·17	1·17	1·11
3.	0·81	0·92	1·00	0·90	0·88	0·84
4.	0·67	0·71	0·80	0·73	0·60	0·64
5.	0·31	0·45	0·48	0·26	0·22	0·23

The probable error of any one of these values is about ±0·02; and they represent the quantity which we are actually seeking, viz. the

apparent blackening, or relative density, of the faint strip at the point where it crosses the fogging strip. This is, of course, what is measured in practice, the difference between this "contrast" and the density of the unfogged image being the advantage gained by fogging.

Any particular strip 1–5 gives a curve of the form shown in fig. 1, the whole five from the one plate being given together there.

The numbers at the sides represent the densities of the unfogged faint images that are being brought out in each case, these densities, as always, being measured relative to the clear (unexposed) parts of the plate.

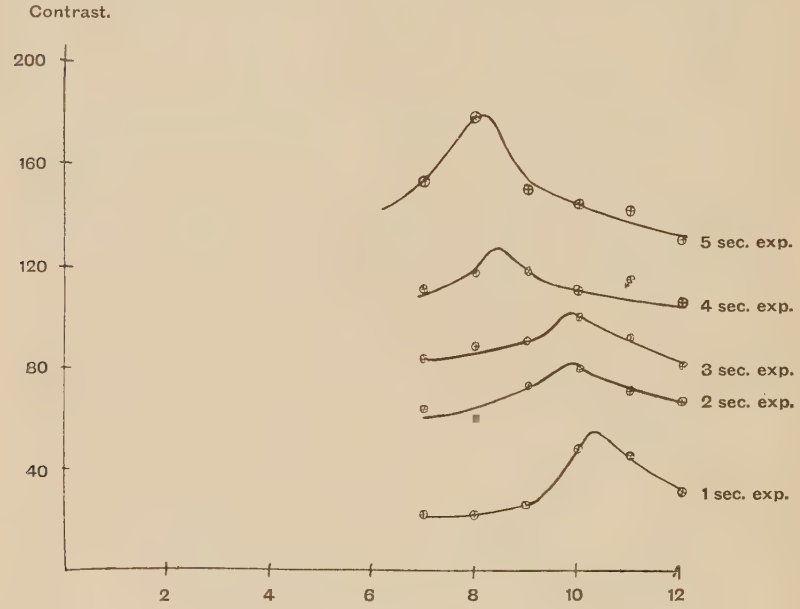

FIG. 1.—Fogging Exposure in Seconds.

From this single set of curves we can make several deductions. In the first place, we have a definite proof, if one were needed, of the actual existence of maxima in the curves, *i.e.* that there actually is an optimum fogging exposure which will bring out the faint image to the greatest possible extent.

In the second place, we see that, whilst the absolute value of the contrast naturally decreases regularly with decreasing density of the faint image, yet the relative value of the contrast increases very greatly.

The curve (fig. 2) showing the ratio of maximum contrast to density of image plotted against density of image brings this out very clearly. The curve evidently approaches the line $x = 0$ asymptotically (there is another point at (1, 51) not shown in the figure), from which we see that, as the images become progressively fainter, the fogging becomes enormously more effective.

Although, however, this ratio becomes so large, yet the absolute value of the contrast, as we have seen, steadily diminishes; and for images fainter than about 0·01 it was not found possible to obtain reasonably smooth curves.

It was also found that to bring out these excessively faint images at all it was necessary to employ very considerable fogging exposures; as indeed we should expect from the curves given above, which show that the position of maximum contrast shifts very appreciably to the right as the image to be brought out becomes fainter. In other words,

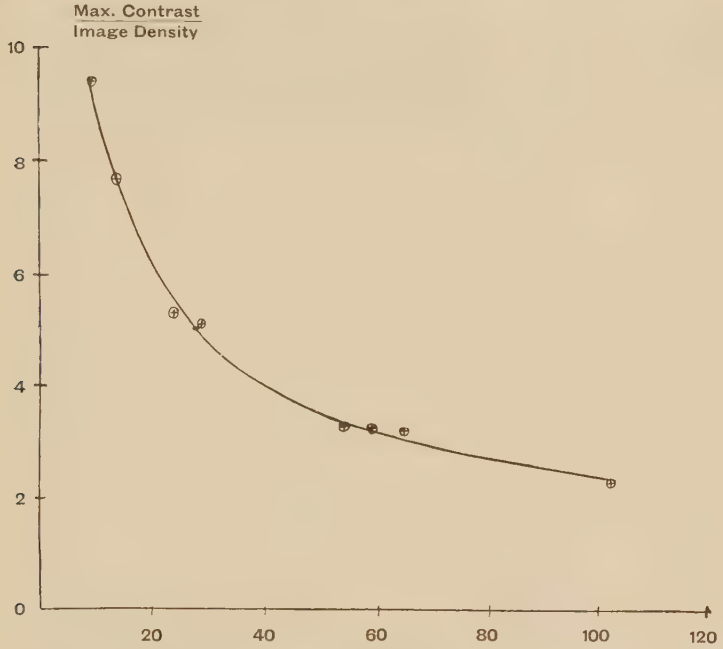

Fig. 2.—Image Density.

the fainter the image is, the longer becomes the "optimum" fogging exposure that will bring it out with the maximum possible contrast.

As a practical point it is important to notice that, whilst the curves are for the most part fairly flat, yet they rise quite sharply to the maximum, so that it is desirable to give the requisite degree of fogging with some accuracy, a difference of 15 per cent. of the exposure giving a difference in contrast of as much as 50 per cent. of the maximum obtainable in the case of the fainter images.

The facts that we have deduced so far were confirmed by taking plates whose exposure times lay in regions well on either side of the maxima. Thus, for example, one plate had the following exposure times—

A.	B.	C.	D.	E.	F.		1.	2.	3.	4.	5.
6^s	5^s	4^s	3^s	2^s	1^s		5^s	4^s	3^s	2^s	1^s

and gave the curves of fig. 3. Another plate, taken with the same intensity of light, but with longer exposures, viz.

A.	B.	C.	D.	E.	F.	1.	2.	3.	4.	5.
9^s	8^s	7^s	6^s	5^s	4^s	5^s	4^s	3^s	2^s	1^s

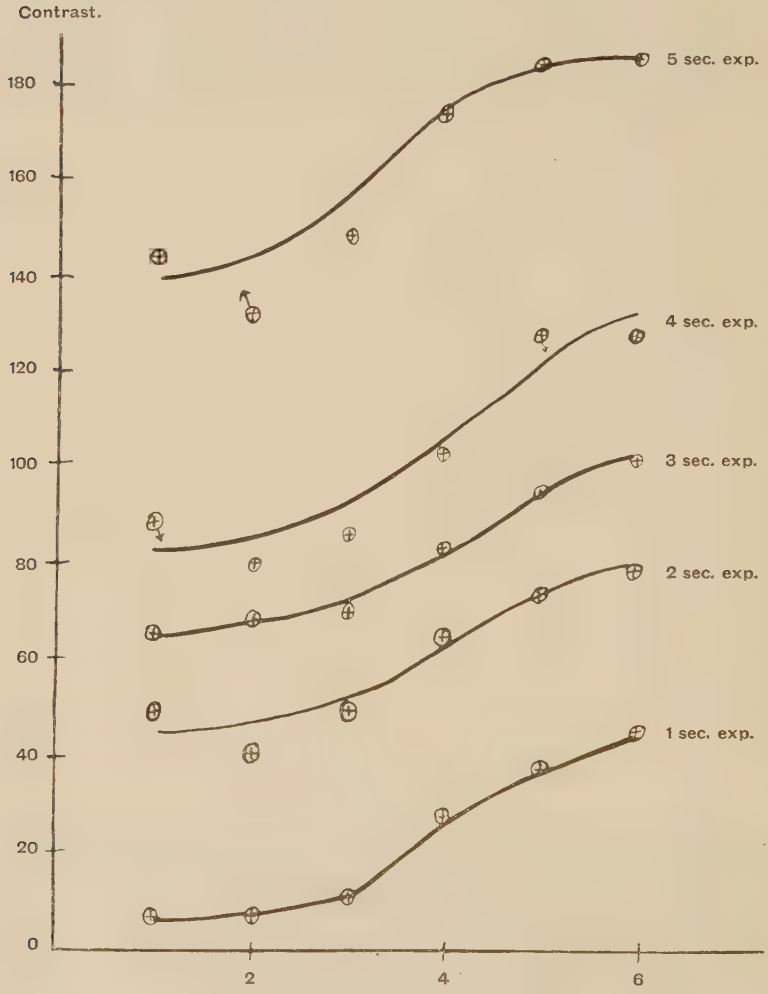

FIG. 3.—Fogging Exposure in Seconds.

gave curves which were similar in shape, but which sloped the other way, showing that the maximum was now passed.

The next question which arises is whether the development affects these maxima in any way, either in magnitude or position, and if so, in what way. Accordingly, a plate was taken with exactly the same

exposure times as those which gave the maxima in fig. 1, but it received a longer development.

The curves obtained (fig. 4) did not resemble those of fig. 1 at all; not only had the contrast increased in absolute value, but the maxima had been driven right off the plate.

It is at this point that we are able to realise the importance of the curves drawn with the fogging densities, instead of the fogging exposure times, as abscissæ. For, drawing the curves of fig. 1 in this way, and putting in beside them the curves for the over-developed plate that we are considering, we obtain fig. 4.

It is obvious from this figure that the actual density of the image, however produced, has far more effect than the exposure time which may happen to have produced this density, in determining the position of the maximum, and, in fact, in settling the character of the curve

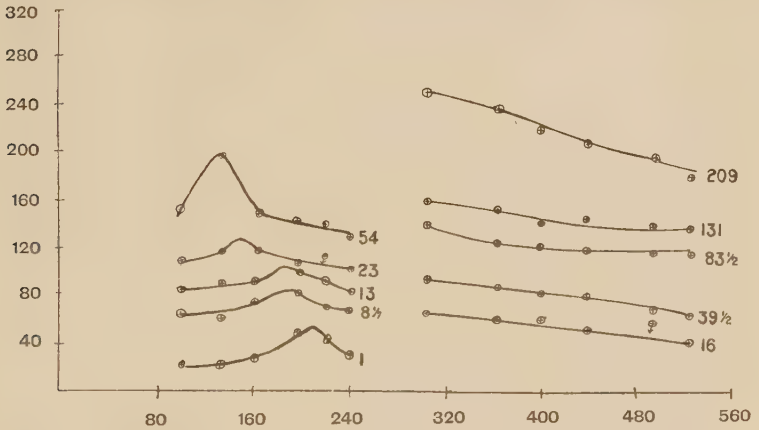

Fig. 4.

generally. Indeed, from the way in which the two sets of curves dovetail into one another, the most natural assumption to make seems to be, that the "optimum" fogging for any given faint image depends only on the density of the image and not at all on its exposure or past history; or, in other words, two images with the same density will give the same curve, whether these images are produced by over-exposure and under-development, or by under-exposure and over-development.

This assumption, if it could be proved true, would be of importance theoretically, for it rules out of court at once any theory which looks to the qualities of the incident light for an explanation of this particular effect, and indicates rather that the explanation must be sought for among the properties of the plate itself.

To test it, a number of plates were taken under circumstances differing as widely as possible, and the results are given all together in fig. 5; this is made up from five different plates, their individual curves being distinguished by the numerals I.–V.:

I. is the plate used in fig. 1, and had exposures 1^s–5^s (image) and 7^s–12^s (fogging).

II. received the same development as I., but had exposures 1^s–5^s (image) and 1^s–6^s (fogging).

III. was developed at a higher temperature than I. and II., but received the same exposure as I.

IV. received the same exposure as II., but was developed for twice as long in developer of twice the concentration, *i.e.* for 16^m in Azol 1 in 50.

V. was also exposed for the same times as II., but the light was much fainter—(0·8 amp. through the lamp); the developer was of normal strength, but the time of development was 12^m.

Thus, taking the images in I. as standard, we see that II. is a normal continuation of I., but that III. corresponds to a normal exposure and an over-development; IV. to an under-exposure and extreme over-development; and V. to extreme under-exposure and an over-development.

We have thus a considerable variety of conditions represented; so much so, that only parts of III. and IV. could be used, the rest falling outside the region where useful results could be obtained.

It will be observed that all the curves fit into one another as if they had been made from a single plate. This is particularly evident in the neighbourhood of the maxima, where the results are most trustworthy. Thus 59 II. fits in between $62\frac{1}{2}$ IV. and 54 I.; 2 V. between 1 I. and 3 II., and at the other end the continuation of $39\frac{1}{2}$ III. would come in between 23 I. and 54 I.; and so on.

The irregularity shown by the last three points of $164\frac{1}{2}$ IV. is not of any great significance, owing to the amount of chemical fog in this region caused by over-development.

Evidently, too, the curves corresponding to image densities greater than about 1·5 are of such a shape as to cut the y-axis before reaching a maximum, and so are not of the type we are dealing with here.

We may fairly conclude, therefore, that for faint images (and strictly speaking, of course, we must add, for Ilford panchromatic process plates exposed to red light and developed in Azol) the optimum fogging density required to bring out the maximum contrast for any given image depends solely on the resultant density of this image, produced jointly by exposure and development. The separate values of the times of exposure and of development do not enter into the question at all, at any rate within a range of development greater than any likely to be required in practice.

The general nature of this result makes it seem at least probable that it will be true for other cases also—for different plates, developers, and wave-lengths—but theoretically it is of importance to have established its truth in even a single instance. For, as has already been said, it shows that the explanation of this fogging effect, which we have been discussing, is to be looked for in the properties of the plate, and not in those of the light falling on it, and in this way it has a general bearing on the whole question of photographic action.

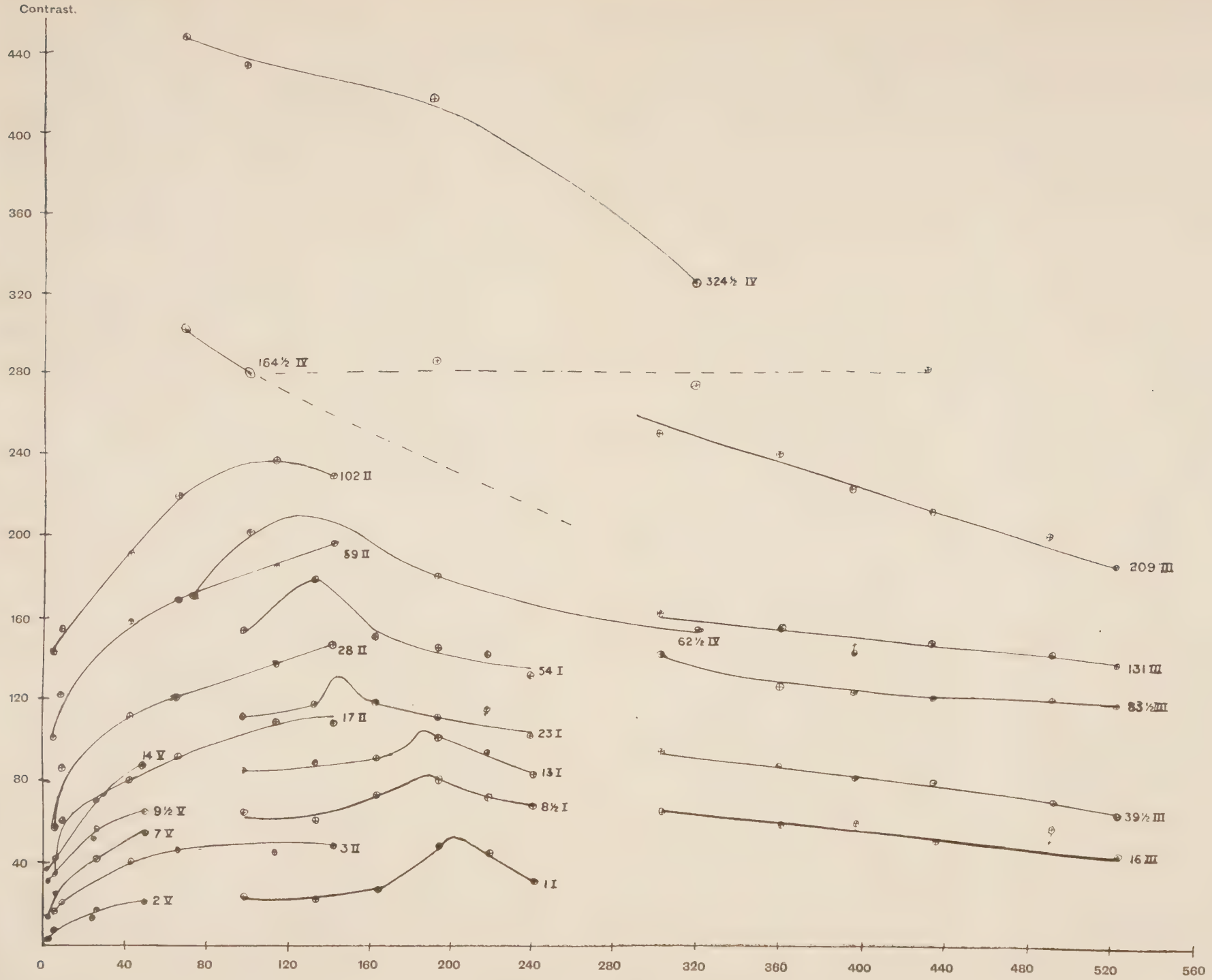

Fig. 5.—Density of Fogging Exposure.

Practically, the steepness of the curve near the maximum is of considerable consequence if the best results are to be obtained. The determination of this and other quantities required in practice, however, must be carried out separately in each particular case. The object of the present paper has not been to do this for itself, but only to describe an accurate method of doing so, and to indicate the nature of the results likely to be obtained. There seems to be no reason to suppose that the broad outlines of the phenomenon will be strikingly different in other cases.

In conclusion, I must express my warmest thanks to Mr. G. M. B. Dobson, M.A., who designed the photo-electric apparatus used in making the measurements, and without whose constant help and advice the investigation could not have been undertaken.

The Proper Motion of a Faint Star near η Cassiopeiæ.

By W. M. Smart, D.Sc.

(*Communicated by Professor A. S. Eddington, F.R.S.*)

In the course of the measurement of the relative proper motions of stars in the region of η Cassiopeiæ, an unusually large proper motion was detected for the faint star whose approximate co-ordinates for 1900·0 are

$$\text{R.A. } 0^h\,45^m\cdot3\,; \qquad \text{Dec. } +57^\circ\,45'.$$

The star was subsequently identified as No. 1 in a list, by Hubble, of 12 faint stars with considerable proper motion (*Astronomical Journal*, 29, 168). The centennial proper motion given by Hubble is 1″·50 in position-angle 72°·2; he states that the accuracy of the measures is not great. The star was at once placed on the parallax programme of the Yerkes Observatory, and the following results, from 11 plates, are quoted from the *Publications of the Yerkes Observatory*, 4, part iii., 2:

Parallax	$+0''\cdot056 \pm 0\cdot008$.
μ_α	$+0^s\cdot193$.

The interval between the first and last plates of the series was 3·1 years. The photographic magnitude of the star is 11·5 according to Hubble, and it is described as having "a decidedly yellowish tinge."

The interval between the pair of plates measured at Cambridge, on which the results below are based, is exactly 18 years. The star is about 30′ from the centre of the plates. The components of proper motion were found to be

$$+154''\cdot8 \quad \text{and} \quad +45''\cdot8,$$

giving the centennial motion, relative to 20 comparison stars, of

$$161''\cdot4 \text{ in position-angle } 73^\circ\cdot5.$$

On the plates are 34 B.D. stars whose mean magnitude on the Harvard scale is 9·3. The relative centennial proper motions of these stars as derived from the measures were employed in conjunction with the mean parallaxes of stars of this magnitude to reduce the relative proper motions to absolute proper motions. The mean parallaxes given by Kapteyn (*Groningen Publications*, No. 29, table 27) were used. In this way the correction, absolute P.M. *minus* relative P.M., was derived:—

In x,	$+1''\cdot1$.
In Dec.,	$-1''\cdot1$.

The components of absolute proper motion (per century) are then:—

$$+155''\cdot9 \quad \text{and} \quad +44''\cdot7,$$

yielding the centennial motion of

$$162''\cdot2 \text{ in position-angle } 74°.$$

The probable errors indicated by the measures of the several images are $0''\cdot6$ per century in each co-ordinate. The motion is thus greater than that attributed by Hubble. In the usual notation, the Cambridge results are:—

$$\mu_\alpha = +0^s\cdot1950.$$
$$\mu_\delta = +0''\cdot447.$$

The proper motion given by the Yerkes parallax measures is relative to 5 comparison stars whose mean magnitude on the Harvard scale is 11. The parallactic motion in R.A. of magnitude 11 stars in this part of the sky is $+0^s\cdot13$ per century. The Yerkes proper motion in R.A. is thus absolutely

$$\mu_\alpha = +0^s\cdot194,$$

a result in exceedingly close agreement with the Cambridge result. Perhaps it is scarcely legitimate to apply this correction in the case of the Yerkes comparison stars—the number is hardly large enough to secure practically the elimination, in the aggregate, of the non-parallactic motions.

The spectrum of the star is not known. Its absolute magnitude (photographic) is about 10·2. Accordingly, it is a dwarf, probably of type M.

The cross-velocity, derived from the Cambridge proper motions and the Yerkes parallax, is

$$137\ (\pm 20) \text{ kilometres per second.}$$

Observatory, Cambridge:
1923 August.

On the Intensity-Distribution in Short Grating Spectra and Objective-Prism Spectra as a Function of Spectral Type and Absolute Magnitude. By Bertil Lindblad, Ph.D.

The present note is a continuation of my "Note to Dr. Knut Lundmark's and Dr. W. J. Luyten's Paper on the Determination of the Colour-Equivalent of a Star." * In a recent "Note on the Determination of Absolute Magnitude from λ_e and λ_m," † those authors have defined more precisely their point of view, which makes it possible to see more clearly where our difference of opinion regarding the matter in question has originated.

1. It was my aim to show in the note mentioned that the systematic difference in qualities between λ_{eff} and λ_{min} in their dependence on absolute magnitude ‡ is in all probability real, and caused by the variation of intensity of the cyanogen band at $\lambda 3883$ Å in stellar spectra.§ In order to show the effect of the band I excluded the stars of $\lambda_{\text{eff}} < 4180$ Å, nearly corresponding to type Go, *because the cyanogen absorption vanishes for earlier types*, as well as every clear indication of a qualitative difference between λ_{eff} and λ_{min}. This is the reason why I said that for the excluded early-type stars "it would be futile to try a discrimination between different degrees of luminosity by means of λ_{eff} and λ_{min}." The statement by Lundmark and Luyten that "it is just for such early-type stars that Lindblad has applied his method of estimating the absolute magnitude from the intensity of the cyanogen bands" is a confusion with quite another method, as can be seen by a study of my paper cited.§

After the explanation given here I suppose that there is no objection to the exclusion of the stars of $\lambda_{\text{eff}} < 4180$ Å. We then put for the remaining 45 stars

$$\lambda_e - 4230 = k(\lambda_m - 3780) + a + b\text{M},$$

where the factor k takes account of the different scales of λ_e and λ_m when treated as colour-equivalents, and the term bM takes account of the possible systematic difference in their dependence on luminosity. For M I have used the spectroscopic absolute magnitudes determined by Adams and his associates in the list of 1646 parallaxes. I find from the 45 equations of condition

$$k = 1{\cdot}15 \pm 0{\cdot}13 \text{ (m.e.)}.$$
$$a = -8{\cdot}8 \pm 5{\cdot}1 \text{ Å}.$$
$$b = 3{\cdot}7 \pm 1{\cdot}2.$$

This value of b is a direct measure of the qualitative difference between λ_e and λ_m which is clearly shown in Plates 2 and 3 of my previous note.

The mean error of unit weight is ± 25 Å, *i.e.* nearly the same as

* *M.N.*, 83, 97, 1922.
† *M.N.*, 83, 470, 1923.
‡ *Astrophysical Journal*, 49, 289, 1919; *Uppsala Universitets Årsskrift*, 1920.
§ *Mount Wilson Contr.*, No. 228; *Astrophysical Journal*, 55, 85, 1922.

that found by Lundmark and Luyten (± 23 Å), applying the formula $\lambda_e = \lambda_m + a + b\mathrm{M} + c\mathrm{S}$ to the whole series of types from B to M, which leads to $b = 1{\cdot}38 \pm 0{\cdot}37$. The low value of b in this case is evidently due to the presence of the early-type stars. This formula is not appropriate for the later types taken alone, because of the irregular variation of λ_e and λ_m with the spectral type S.

It seems fair to assume that part of the mean errors of unit weight given above is due to the fact that the linear formulæ used probably give a rather poor representation of the actual relations. The accidental mean errors of λ_e and λ_m, supposed to be equal, ought then at least to be less than ± 18 Å.

The magnitude of the coefficient b found here is in agreement with the effect that can be expected in $\lambda_{\min}$ from the known variation of intensity of the cyanogen band at $\lambda 3883$ for stars of different luminosities within types G–K5. It is sufficiently great to make a discrimination of giants and dwarfs by means of λ_{eff} and $\lambda_{\min}$ alone not an entirely hopeless task, if sufficient precaution against the influence of systematic errors is taken; but it is not my intention to advocate the practical use of the method in this form.

2. The effective wave-length is for the later types essentially dependent on the marked contrast in intensity between the two parts of the photographic spectrum separated by the G-band. The increase of this contrast proceeds much more rapidly with the spectral type than can be allowed for by the simultaneous change of temperature according to Planck's law. I therefore form a *local colour-equivalent* by comparing in short and narrow objective-prism spectra the intensities immediately adjacent to the G-band, namely, the region $\lambda 4227$–G with G–$\lambda 4383$. The comparison is made by means of a measurement of the exposure-ratio in the way described in a previous paper (though applied to other spectral regions).* The character of the logarithmic exposure-ratio for the spectral regions just mentioned as a real colour-equivalent, in spite of the nearness of the two compared regions, is shown by fig. 1, where I have given the preliminary results for a few stars in the Mount Wilson catalogue of 1646 parallaxes, which have been photographed in connection with a larger work on the proper-motion stars in the Greenwich polar zone. Circles denote giants, black dots dwarfs; the ordinate is the logarithmic exposure-ratio. The resulting curve may be compared with Seares'† curve for the exposure-ratio of the blue and yellow regions of the spectrum, as well as with diagrams containing the effective wave-length as function of spectral type. For types earlier than F5 the presence of Hγ seriously disturbs the estimation of the exposure-ratio.

The intensities in the cyanogen regions may be measured in the same way by comparison with the intensity of an adjacent region of the continuous spectrum. For the $\lambda 4216$ band I obtain the exposure-ratio between the region $\lambda 4144$–4184 Å in the densest part of the cyanogen band and the region $\lambda 4227$–4272 Å as the standard region. The funda-

* *Mount Wilson Contr.*, No. 228, p. 3.

† *Proceedings of the National Academy of Science*, 5, 232, 1919.

mental properties of this exposure-ratio have been discussed in the paper just cited. If the minimum wave-length is perceptibly affected by the λ3883 band, it ought to show something of the characteristics of this exposure-ratio, which measures the degree of absorption in one of the bands. A comparison between λ_{eff} and λ_{min} ought therefore to have some analogy with a comparison between the two exposure-ratios, of which one represents the contrast at the G-band, and the other the degree of absorption in the cyanogen band at λ4216. I shall call these

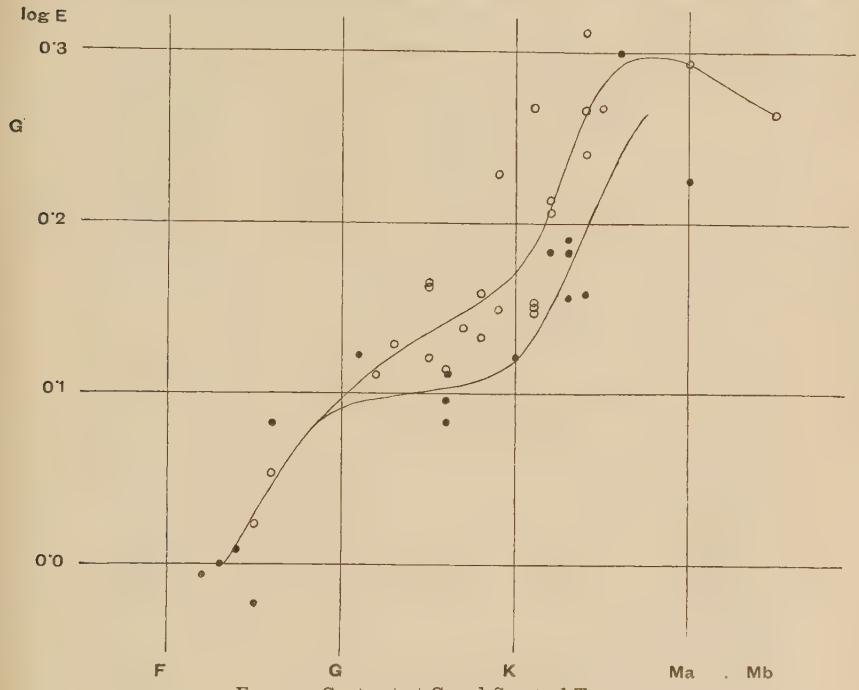

Fig. 1.—Contrast at G and Spectral Type.

○ M<3.
● M≥3.

exposure-ratios "G" and "C" respectively. Fig. 2 gives "C" as function of spectral type and absolute magnitude for the spectral types Go–Mb. The diagram is of the same character as that reproduced on p. 29 of my Mount Wilson paper, which contains a more detailed study of the cyanogen effect. The rapid decrease of "C" with advancing type in the "dwarf" series seems to be caused by an absorption effect in the standard region λ4227–4272 rather than by decreasing cyanogen absorption in the region λ4144–4184. The region λ4227–G has a remarkably lower intensity than neighbouring regions for "dwarfs" of types K5–M. The effect may be partly due to an increased unsymmetrical extension of the wings of the very strong line λ4227 of Ca with decreasing luminosity and advancing type.

Eliminating the spectral type, we get the diagram fig. 3, which still shows a very clear separation of the two luminosity-classes on the basis of a measurement of "C" and "G" alone. Table I. gives the observed

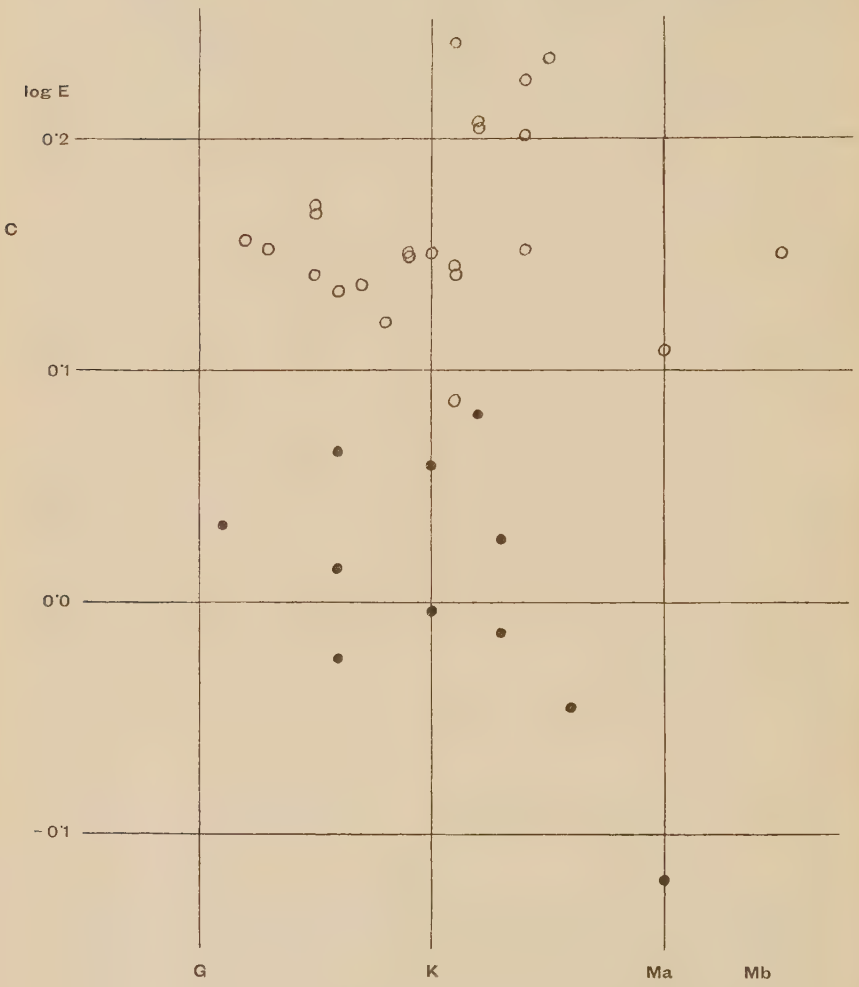

FIG. 2.—Cyanogen Absorption and Spectral Type, discriminating "Giants" and "Dwarfs."

data in the case of those stars of type Go or later for which both "C" and "G" have been measured. n_1 and n_2 are the numbers of images used for deriving the exposure-ratios. Spectral type is the "estimated" type given by Adams and his associates in the list of 1646 parallaxes. The absolute magnitudes are from the same source.

TABLE I.

Boss, Cincinnati, or Groombridge No.	Harvard Vis. Mag.	Spectrum.	M.	"G" log E.	n_1.	"C" log E.	n_2.
B86	6·4	Ko	3·9	0·121	12	0·059	13
Gr 127	6·9	G7	1·4	0·138	5	0·137	4
B349	5·5	G5	−0·3	0·120	6	0·171	8
B486	6·2	G6	−0·2	0·113	4	0·134	4
B1758	4·8	K5	0·3	0·267	4	0·234	5
B2161	7·0	G6	5·8	0·096	4	0·065	4
B2195	3·8	K4	0·4	0·240	5	0·201	4
B2234	5·2	G5	−0·4	0·164	5	0·141	4
B2294	6·1	K2	0·7	0·213	9	0·204	8
B2338	6·1	G3	−1·0	0·128	5	0·152	5
B2348	4·2	G5	−1·3	0·162	6	0·167	5
B2378	6·3	Mb	0·8	0·263	2	0·150	2
B2380	6·1	K2	5·7	0·183	7	0·081	5
B2384	6·6 6·1	Ko G9	0·9 0·0	0·228	3	0·150	4
C1050	6·7	G1	4·2	0·122	5	0·033	4
C1218	6·8	K6	8·3	0·299	4	−0·045	4
B2899	3·9	K1	1·4	0·153	3	0·087	4
B2910	5·2	K1	0·7	0·147	4	0·141	4
B2921	6·2	Ma	1·0	0·293	5	0·108	5
B2935	7·6	Ma	10·5	0·225	5	−0·120	4
C1627	6·4	G6	5·3	0·113	2	0·015	2
B3381	6·0	G8	0·8	0·132	3	0·120	3
C1712	6·6	K3	6·4	0·183	3	0·027	3
B5280	6·4	K4	0·8	0·266	5	0·225	5
C2709	7·9	G6	5·3	0·083	3	−0·024	4
B5486	6·2	K4	0·5	0·311	10	0·152	9
Gr 3813	7·9	K1	1·7	0·267	5	0·240	2
B5966	4·7	G2	−0·2	0·110	3	0·156	2
B6078	3·4	K1	2·3	0·150	6	0·144	4
Gr 4129	8·0	G9	2·5	0·149	11	0·149	9
B6129	6·6	K3	6·7	0·190	7	−0·013	8
Gr 4170	8·2	K2	1·4	0·206	3	0·207	3

Figs. 1–3 of this paper are to be compared with figs. 1–3 and 6, 7, 9 respectively of my papers on the measurement of λ_{eff} and λ_{min} cited above at the beginning of § 1. It is of importance to notice in connection with the discussion of § 1 that *"G" and "C" in their relation to each other and to spectral type show an accentuation of a qualitative difference already discernible between* λ_{eff} *and* λ_{min}.

The analogy between "G" and λ_{eff} seems to be nearly perfect for types later than Fo. The rate of change of the exposure-ratio with spectral type is very remarkable. The rate is almost as rapid for the exposure-ratio of the regions adjacent to G with centres at a distance of

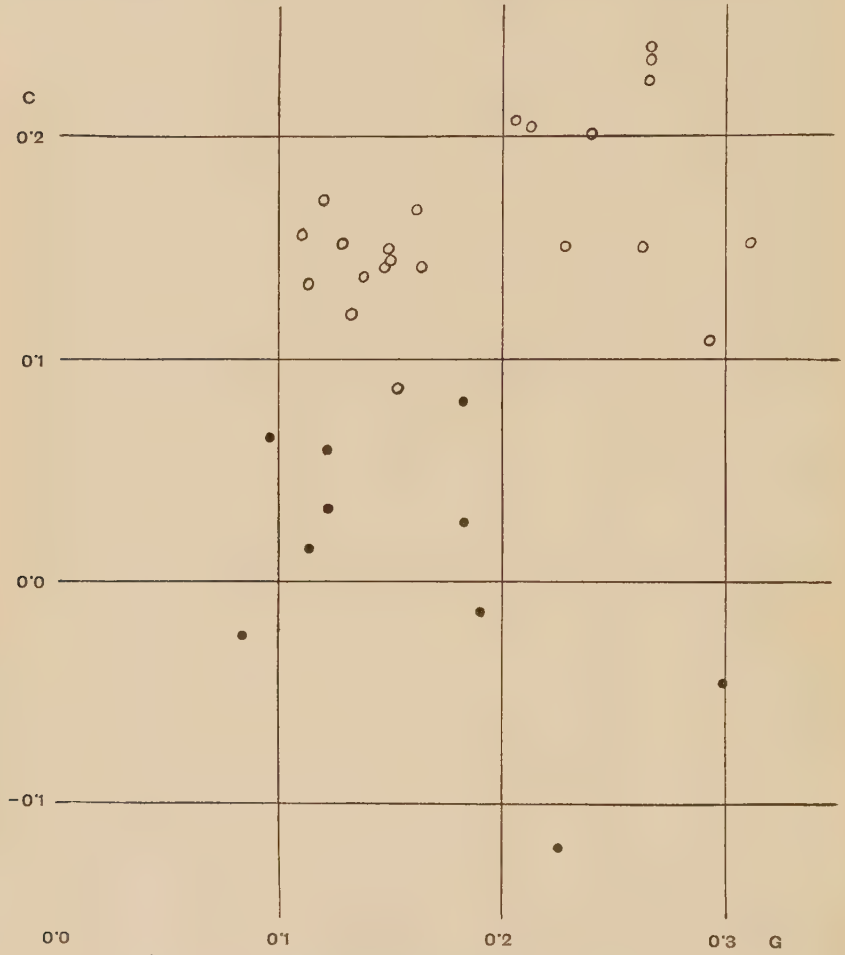

FIG. 3.—Cyanogen Absorption and Contrast at G for the Two Luminosity-Classes.

only 80 Å from each other as for the exposure-ratio formed by Seares for the blue and yellow regions with a distance between the centres of the regions amounting to about 1000 Å. The rapid change at G cannot be explained by a direct application of Planck's law. In a recent paper on solar radiation * I have given some reasons for believing that the

* *Nova Acta Reg. Soc. Sc. Upsaliensis*, ser. iv., 6, No. 1, 1923.

FIG. 4.—Reduced Proper Motions for "Giants" and "Dwarfs," separated by Spectro-Photometric Analysis.

"photospheric" radiation of the sun has a sharp peak near G with a very abrupt fall towards the short wave-lengths. From the evidence of stellar spectra given here it seems as if for types a little later than that of the sun the abrupt fall mentioned is located as a complete break exactly at the G-band and increases rapidly with decreasing effective temperature.

λ_{min} undoubtedly shows some characteristic traces of the cyanogen absorption between λ3840–3883 Å. It is true that the values of λ_{min} are somewhat lower than the wave-lengths of the band, but because of the extremely low dispersion of the grating spectra, leading to a great impurity of the spectra, the apparent extension of the border of the spectrum images towards short wave-lengths, which is measured by the value of λ_{min}, is dependent on the intensity distribution within a wide range of the spectrum. But λ_{min} has evidently also some qualities in common with λ_{eff}. We can see the cause of this immediately. λ_{min} was reduced to a certain value of the diameter of the central image, which is not far from equivalent to a certain intensity near the effective wave-length in the spectrum. The value of λ_{min}, therefore, results from a comparison between the intensity near λ3800 with the intensity near λ4200. The compared regions are thus so widely separated that the general colour-effect of the photographic spectrum combines with the effect of the cyanogen absorption. In the case of "C" as measured here the general colour-effect has been eliminated.

In fig. 4 is given an illustration of the results of a spectrophotometric discrimination between "giants" and "dwarfs" for proper-motion stars in a small area of the Greenwich zone. The area in question is situated between the approximate limits R.A. $21^h\ 50^m$–$0^h\ 10^m$, Decl. 72°–78°, and is covered by two partly overlapping plates taken with the Zeiss-Heyde astrograph of the Upsala Observatory. The proper motions were reduced to apparent phot. mag. $8^m{\cdot}0$, and are represented by crosses in the figure. The diagram treats "dwarfs" and "giants" of types G0–K2 separately. For this discrimination the degree of cyanogen absorption was used, and to some extent also the intensity-distribution near the silicon line at λ3906.* The figure indicates that the evidence from the spectrophotometric work is reliable for predicting luminosity-class.

With the dispersion of the objective-prism spectra used here (1·4 mm. between Hγ and Hε) and an aperture of 15 cm. narrow spectra of stars of the tenth photographic magnitude can be obtained in about one hour. This means that the objective-prism spectra do not require a much longer exposure-time than well-exposed grating spectra, and that the objective-prism spectra are therefore infinitely superior for the treatment of the problem in question. A study of "C" and "G" in the way described here will also be possible with still smaller dispersion of the spectra.

Astronomiska Observatorium, Upsala:
1923 *September* 4.

* Cf. *Mount Wilson Contr.*, No. 228, p. 18.

The Total Solar Eclipse of 1922 *September* 21. *Observations by the Sydney Observatory Expedition at Goondiwindi, Queensland.* By W. E. Cooke.

1. To test the Einstein theory the astrograph was mounted at Goondiwindi on two pillars; the south was constructed of well-seasoned timber on a concrete base, and the north was a specially made iron casting. The observatory was built of wood and galvanised iron, with a meridian opening in the roof extending from the zenith to the Pole, and a large opening in the western wall, protected by a stout canvas blind.

Imperial S.S. whole-plates were used, each in a special holder which had been carefully "squared-on." Eight exposures were given, each of 10^s duration, and plates 1, 3, and 5 were also exposed to a "test" region situated exactly 5° south of the eclipse region and of the same R.A. In order to do this smoothly and rapidly suitable stops were placed on the large declination circle.

For a driving clock a Grubb chronograph mouse-controlled clock was loaded up sufficiently to carry the heavy weight of the astrograph. It was not altogether a satisfactory arrangement, but the best we could manage. During the preliminary task of taking the comparison plates it worked beautifully, the star's image remaining exactly bisected for minutes. Unfortunately it partially failed at the critical moment, and we only succeeded in obtaining two eclipse plates (Nos. 1 and 4) with steady clock-drive. These two might have given valuable results if the definition had been satisfactory, but it was bad, and the star-images were too diffused for such very delicate work. A considerable time has been spent over the examination and measurement of the plates, but for the intended purpose the results are valueless.

I think, however, that the time and money spent over this portion of the expedition have not been wasted Astronomers are doubtless interested in the possibilities of the astrograph for this class of work, and will be looking for some evidence of its usefulness or otherwise in such a capacity. Its comparative failure at Principe and Sobral was ascribed to the form of mounting, and better results were expected this time. From press reports I gather that the weather at Christmas Island was so bad as to prevent the carrying out of the prearranged programme. At Goondiwindi the sun's altitude was rather small, therefore it would not be fair to condemn the type of instrument on account of its several failures. We had a number of good evenings prior to the eclipse, and its performance under those conditions is worth recording.

Goondiwindi is on an extensive bush-covered plain. To the naked eye the sky is superb, stars of the 3rd and 4th magnitude being plainly visible almost down to the horizon, with scarcely a sign of flicker. Through the telescope, however, a very decided change in the seeing occurred at about 40° altitude. At about 45°–50° altitude we frequently had good, but never very good, definition. The best results were obtained by stopping the aperture down to 4 inches, when good images

were produced over a field of 6 inches in diameter. We found 5^s sufficient exposure for stars 1, 5, 12, and 17 of the eclipse field (see *Monthly Notices*, 1920 May, p. 629), but for the 9th magnitude stars 6, 7, 13, and 14 this time had to be so greatly increased that we decided to limit ourselves to the production of good images for the four stars 1, 5, 12, and 17 only. At the true eclipse altitude we found conditions altered for the worse. It was impossible to obtain a measurable image of No. 12 with the aperture and exposure previously used, and we were compelled to increase the aperture to 7 inches and the exposure to 10^s. None of the comparison plates were very good: one was good, but most of them were only fair.

The impression finally remaining in my mind is this: That in future expeditions it is not worth while to attempt this particular type of work with the astrograph, nor at an altitude less than 50° (or even 60°).

2. A photoheliograph was mounted on an old 6-inch Grubb equatorial stand, and provided with a focal plane shutter. The 4-inch object-glass was stopped down to $1\frac{1}{2}$ inches. A number of photographs of the partial phase were taken at intervals of 2^m and, near the four contacts, of 20^s. Definition was poor. Measures were made, by the help of an astrograph réseau, of (1) the polar diameter, (2) the equatorial diameter, (3) the N.S. component of the distance between the "horns," (4) the E.W. component of the same distance.

These measures are given below. Estimates of the times of first and last contacts were made by the method suggested by Mr. Innes in *Union Observatory Circular* No. 45, the principle being that the time from tangential contact varies as the square of the chord between the horns. For 2nd and 3rd contact another form of computation was necessary.

If M and S are centres of moon and sun respectively, and C the centre of the chord AB, joining the horns, MSC are in a straight line and MC perpendicular to AB. Let MC continued cut the moon's limb in J and the sun's in H, then the time is proportional to JH. Omitting plate No. 1, which was greatly overdeveloped and very dense, and using only the four or five photographs nearest the contacts, we obtain the times of contact, both by graph and computation, which are given below. Probably the time of first contact is less reliable than the others. On the whole we may say that the phenomenon occurred about 12^s earlier than predicted from the quantities given in the *Nautical Almanac*.

To those who make similar observations in future I suggest:

(1) Take special care to secure uniformity in development. Start with one or two of the plates that are not very critical.

(2) Reduce the intervals near contacts to, say, 10^s.

I desire to call attention to one remarkable feature of these photographs. All exposures were on similar plates and taken in a similar manner. Apart from slight variations in development, all showed the same quality of limb. Plates 38 and 43, however, were absolutely blank. The former was taken 4^s before and the latter 2^s after

totality. The exposures were made according to a prearranged programme, and, even supposing some error may exist in the computed times of contact there is no doubt possible, from the sudden outburst of light and shouts of surrounding spectators, that No. 43 was taken about 2^s after the sun's reappearance. I recommend this peculiarity to the attention of future observers, and suggest that arrangements be made, if possible, to take several photographs at 1^s interval round about the times of 2nd and 3rd contacts.

Photographs of Partial Phase. Summary N.S.W. Standard Times.

Times of	Predicted.	Observed. By Eye.	Observed. By Graph.	Observed. By Computation.	Obs. – Pred.
	h m s	h m s	h m s	h m s	s
1st cont.	3 05 31·5	3 05 30	3 05 01	3 05 06·0	−25·5
2nd ,,	4 11 47·7	...	4 11 34·8	4 11 34·7	−13·0
3rd ,,	4 15 18·5	4 15 08	4 15 09·4	4 15 08·2	−10·3
4th ,,	5 14 49·8	5 14 27	5 14 37·3	5 14 37·6	−12·2

The figures "by computation" were those adopted as the observed times. The 2nd contact by eye was lost through failure of stop-watch.

3. The old Reichenbach "repeating circle" of Sir Thomas Brisbane was cleaned up, provided with new telescope and transit micrometer, and converted into an almucantar instrument for the determination of time and latitude. The results further corroborate the value of this method for field work.

The services and private wireless establishment of Mr. P. Shaw were very kindly placed completely at our disposal. Special signals were sent from Washington, via Annapolis and also New York, for several evenings, but "static" was so particularly bad that we were unable to receive these.

Our longitude results depend therefore upon rhythmic signals transmitted by the Sydney radio station and registered at Goondiwindi and the Sydney Observatory. The adopted values, using the *Nautical Almanac* figures for Sydney Observatory, are:

	h	m	s	
Longitude of Goondiwindi station . .	10	01	13·07	E.
Latitude ,, ,, . .	28	32	46·9	S.

There is reason to believe from recent signals transmitted by Lyons and received at Greenwich, Paris, and Sydney, that the longitude of Sydney given in the *N.A.* is about $0^s·5$ too great, and that a better value would be $10^h\ 04^m\ 49^s·0$. Using this in preference, the longitude of Goondiwindi observing station becomes $10^h\ 01^m\ 12^s·5$.

4. Special arrangements were made for the observation of shadow-bands. Mr. Graham of the Sydney Observatory, and Dr. Thomson, secretary and founder of the Royal Geographical Society of Australasia, provided with accurate timekeepers, concentrated their attention upon a white sheet laid on the ground. Contrary to expectation, the shadow-bands were not seen until after 2nd contact. The observers agree within half a second as to the time when they first became visible, viz.

$4^h\ 12^m\ 07^s{\cdot}5$ (Standard Time), or about 33^s after commencement of totality. They also agreed as to the time of the second series, viz. $4^h\ 14^m\ 35^s$, or 33^s before end of totality. The observers also agreed very closely in their determination of the azimuths in which the shadow-bands lay, the first series being N. 34° E., and the second N. 50° E.

These observations, taken by two careful observers who concentrated their attention upon the one object, agree closely amongst themselves, but differ radically from the rather vague reports of the general public.

In future eclipses it is suggested that this phenomenon be observed by trained observers in groups of three, one taking a broad view of the landscape, another looking down on a large white sheet from a slight elevation, and a third situated right alongside the sheet; but it is specially important that each should be provided with a timekeeper rated to one second, and should record definite times, rather than relation to totality.

5. An attempt was made to locate the edge of the moon's shadow on the earth at Grafton, New South Wales, and Beaudesert, Queensland.

A position on the locus of the shadow-edge at Grafton was computed from the *Nautical Almanac*, viz. lat. 29° 40′ 30″ S., long. 152° 56′ E.

Two separate sets of observers provided two distinctly different paths for the shadow-edge. One set placed the observed shadow-edge on the river itself, just south of the city, in latitude 29° 41′ 35″, the other placed it just north of the grand-stand, say 29° 40′ 20″. The disputed strip is a little over a mile, and just includes the whole city of Grafton. There is no doubt that the shadow did not reach the south bank of the river.

Both parties agree in the following points:

Throughout the disputed zone the black moon was completely visible, the corona was visible for the complete circle, though more pronounced on the northern edge; and a bright spot of apparent sunlight persisted in the position of 11 o'clock.

One additional observation seems to decide the matter, viz. that the actual edge of the shadow was plainly seen *along the river*, near its northern bank, by people situated on both banks; that the portion near the city was dark, whereas that near South Grafton was quite bright all the time.

I consider therefore that we may conclude that the edge of the moon's shadow crossed the meridian of 152° 56′ in latitude 29° 41′ 35″.

6. *Shadow-Bands at Grafton.*—Special observations were taken at a place a few miles north of the city, and therefore well within the shadow. Owing to the absence of reliable time observations I cannot say whether they confirm the Goondiwindi results or not. The first series was moving from south to north, slowed down, and became stationary for about 20^s, then reversed their direction. The bands lay in a general E. and W. direction. It appears that during the "stationary" period they must have been moving in a direction parallel to themselves.

The shadow-bands were plainly visible all over Grafton down to the edge of the moon's shadow, but were not observed in South Grafton, *i.e.* just outside the shadow.

It appears to me that the region near the edge of the shadow on the earth should not be neglected by professional astronomers in future, as it seems to have been hitherto. Certain phenomena which crowd one another into the period of a few seconds in the central line of totality are here spread out over minutes.

7. *The Shadow-Edge at Beaudesert.* — This is a small township terminating a short railway running almost south from Brisbane, Queensland. The computed position of the shadow-edge happened to pass exactly over the railway station. The shadow must have passed more than a mile south of its computed position. Two observers went a few miles further south, to an elevated spot known as Mount Mahomet, and there observed totality lasting for $1^m\ 10^s$. The exact locality of Mount Mahomet, and even of Beaudesert, is not very accurately known, but the Surveyor General of Queensland places the spot where the above observations of totality were made at 28° 06′·7 S., and 153° 00′·7 E.

The net results of the observations agree in locating the shadow of the moon on the earth a little more than one mile south of the position computed from the figures of the *Nautical Almanac*; the width of the shadow appears to have been practically the same as predicted; and the time of the phenomenon at Goondiwindi about 12^s before the expected time.

On the Recent Light Changes of Beta Ceti. By A. D. Ross, M.A., D.Sc., F.R.S.E., and R. D. Thompson, M.A., M.Sc.

A cablegram was published in Perth on February 28 announcing the outburst of a star in the constellation of Cetus. The cablegram did not specify the star in question, and the natural inference was that it referred to Mira (o Ceti), which was due at its maximum about March 1. Observations made on the evening of February 28 showed that Mira was then normal, being little above third magnitude. Beta Ceti was seen, and although it was not carefully examined, it certainly could not have been more than half a magnitude above normal (2 2). A further cablegram received on March 1 made definite reference to β Ceti, and since then the authors have kept a close watch on the star on every available opportunity. Fortunately, the meteorological conditions on many evenings in March were excellent. The authors invariably made independent determinations of the magnitude, and, with the one exception of March 13, when the separate determinations were 2·2 and 2·5, the values never differed by more than 0·2 magnitude, while the average difference was under 0·1 magnitude. As a rule, one set of observations was made from a point 2 miles south of Perth, and the other from stations 2 to 6 miles west of Perth. In this way interference from any smoke haze of the city was avoided. By the middle of March β Ceti was at a low altitude at twilight, and a series of tests was carried out on α Phœnicis—a star of similar spectral type—to serve as a guide as to the amount of atmospheric absorption.

The following magnitude determinations were obtained, the second decimal being obtained merely in averaging the two independent estimates:—Mar. 1, 1·75; Mar. 2, 1·9; Mar. 3, 2·0; Mar. 4, 2·1; Mar. 6, 1·95; Mar. 7, 2·1; Mar. 8, 2·2; Mar. 9, 2·35; Mar. 11, 2·15; Mar. 13, 2·35; Mar. 15, 2·2; Mar. 18, 2·3; Mar. 19, 2·35; May 3, 2·25. Beta Ceti was seen also on March 22, but owing to its proximity to the sun, and the presence of some cirrus, it was impossible to give a reliable estimate of its magnitude. Cloudy weather during the latter half of April and at the commencement of May prevented any observations being obtained until May 3.

The list which follows gives, with their respective magnitudes, the comparison stars employed during March in the determination of the brightness of β Ceti. In the beginning of March the altitude of Cetus was just under 20° at the time of observation, and approximate values of the corresponding altitudes of the comparison stars are stated in parentheses:—γ Orionis, 1·7 (51°); ϵ Orionis, 1·7 (59°); β Tauri, 1·8 (28°); ζ Orionis, 2·0 (60°); β Carinæ, 2·0 (43°); β Aurigæ, 2·1 (13°); ϵ Carinæ, 2·1 (51°); δ Velorum, 2·2 (51°); α Arietis, 2·2 (13°); α Phœnicis, 2·4 (26°); δ Orionis, 2·5 (58°); θ Aurigæ, 2·7 (21°); α Ceti, 2·8 (36°); ι Aurigæ, 2·9 (24°). The determination on May 3, with β Ceti at an altitude of about 28°, was made with the following comparison stars:—α Gruis, 2·2 (62°); β Gruis, 2·2 (57°); α Phœnicis, 2·4 (40°); α Pegasi, 2·6 (28°).

The figures given above would appear to leave no room for doubt that a remarkable disturbance took place in the star prior to March 1. If the light curve is plotted, it will be found that the brightness fell off steadily until March 4, after which there appear to be fluctuations with a period of about 5 days, the subsidiary maxima being attained about March 6, 11, 16. It is possible that these small apparent fluctuations are due to observational errors, but the close agreement between the independent determinations obtained through the excellence of the meteorological conditions makes it probable that they are real.

University of Western Australia,
Perth. 1923 *May* 3.

Sydney Observatory, Occultation of Saturn, 1923 *May* 26.

This occultation was observed at the Sydney Observatory through the 11¼-inch Equatorial, with a power of 200. Observer H. E. Rayner. The definition was fair to poor, and scattered clouds interfered.

G.M.T. of Disappearance.

	h	m	s
First contact of ring . .	3	19	31·6
Last „ ball . .		20	29·8
„ „ ring . .		20	55·8

The moon was covered by a cloud at time of reappearance.

LIST OF ADDITIONS

TO THE LIBRARY

OF THE SOCIETY

JUNE 1922 TO JUNE 1923.

Serial Publications received in the regular course are not included in this Annual List.

An asterisk (*) indicates that the work is an excerpt.

Aitken (John):

Collected Scientific Papers. Edited for the Royal Society of Edinburgh by Cargill G. Knott.
(*Royal Society of Edinburgh.*) 8vo. Cambridge, 1923

Algiers, Observatoire:

Carte photographique du Ciel. Zones +1°, +3° (30 charts).
(*Observatory.*)

Allegheny Observatory:

Publications. Vol. 6, Nos. 4, 5.
(*Observatory.*) 4to. Lancaster, Pa., 1920–21

American Association of Variable Star Observers:

*Monthly Report, Nos. 1–10, 1921.
(*Association.*) 8vo. Northfield, Minn., 1922

Anding (E.):

——: Relative Schwere-Messungen in Bayern im Jahre 1902.
(*Gotha Observatory.*) 4to. München, 1912

——: *Système de Référence et Mesure du Temps.
(*Author.*) 8vo. Leipzig, 1913

——: Astronomische Messungen auf dem Bayerischen Hauptdreieckspunkt Asten.
(*Gotha Observatory.*) 4to. München, 1915

Anding (E.):
——: Zehn Breiten zweiter Ordnung.
(*Gotha Observatory.*) 4to. München, 1918
——: Ein didaktisches Hilfsmittel zur sphärischen Astronomie.
(*Author.*) 4to. Kiel, 1919

Aristotle:
De Cælo. De Generatione et Corruptione. Translated into English by J. L. Stocks.
(*Purchased.*) 8vo. Oxford, 1922

Astronomische Abhandlungen:
Ergänzungshefte zu den Astronomischen Nachrichten. Band 4, Nos. 1–6.
(*Editor.*) 4to. Kiel, 1922

Bakhuyzen (H. G. van de S.):
Rapport sur l'activité de l'Association Géodésique pendant la période 1912–1922.
(*Author.*) 4to. Leyden, 1922

Barley (Alfred H.):
The Drayson Problem: His Important Astronomical Discovery.
(*R. A. Marriott.*) 8vo. Exeter, 1922

Barnard (E. E.):
——: *On the Light Changes and other Peculiarities of Nova Persei No. 2 (Anderson, 1901).
(*Author.*) 8vo. London, 1922
——: *Some Peculiarities of the Novæ.
(*Author.*) 8vo. Philadelphia, 1922

Bassermann-Jordan:
Die Geschichte der Zeitmessung und der Uhren.
(*Purchased.*) 4to. Berlin, 1920–23

Band 1, Lieferung B. L. Borchardt, Altägyptische Zeitmessung.
Band 1, Lieferung F. K. Schoy, Gnomonik der Araber.

Bell (Louis):
The Telescope.
(*Publishers.*) 8vo. London, 1922

Bordeaux, Observatoire:
Carte Photographique du Ciel. Zone +16° (2 charts).
(*Observatory.*)

Bos (W. H. van den):
The Orbit of Bu 6832=Σ1834.
(*Amsterdam Academy.*) 8vo. Amsterdam, 1921

Brahe (Tycho):
Tychonis Brahe Dani Opera Omnia: editet J. L. E. Dreyer. Tomus 5, pt. 2.
(*Editor.*) 4to. Hauniæ, 1923

British Association for the Advancement of Science:
Report of the Meetings, 1919, 1920, 1921, 1922.
(*Purchased.*) 8vo. London, 1920–23

Brooks (H. Jamyn):
The Spacial Ether.
(*Author.*) [Privately printed], 1923

Buisson (Ch.) and H. Fabry:
——: *Sur le déplacement des raies solaires sous l'action du champ de gravitation.
(*Authors.*) 4to. Paris, 1921
——: *Mesures de longueurs d'onde dans l'extrémité ultra-violette du spectre solaire.
(*Authors.*) 8vo. Paris, 1921
——: *Étude de l'extrémité ultra-violette du spectre solaire.
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Dr. R. T. A. Innes. Nine miscellaneous slides.

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(*Prof. W. W. Campbell.*)

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Transparency of the Moon, by Warren de la Rue.
(*Mrs. Grove-Hills.*)

Globe of the Moon, by J. Russell, 1797.
(*Miss Bacon.*)

INDEX

PAGE

Adams, W. S., and G. Strömberg, Stellar velocity and absolute magnitude: note on a paper by Professor Eddington and Miss Douglas 474
Adelaide Observatory, Report of the proceedings of the, 1921–22..... 282
Annual General Meeting, Report of the proceedings of the..... 219
Antoniadi, E. M., Note on a paper by M. Shajn 83
Assets and present property of the Society..... 230
Assistant Secretary, Appointment of 231
Astrographic Catalogue, Fourteenth note on the number of faint stars with large proper motions in the Oxford, F. A. Bellamy..... 67
——————————— The stellar magnitude scales of the. Seventeenth note: the San Fernando magnitudes for $-3°$ and $-5°$, and the Paris magnitudes for $+22°$, $+23°$, and $+24°$, H. H. Turner..... 385
Auditors of the Treasurer's accounts for 1922, Report of the..... 224

Barrell, H., Wave-lengths of silicon lines occurring in the spectra of B-type stars..... 322
Baxandall, F. E., Lines of unknown origin in various celestial spectra..... 166
Bellamy, F. A., Fourteenth note on the number of faint stars with large proper motions on plates of the Oxford Astrographic Catalogue for zones $+25°$, $+26°$, and $+27°$..... 67
——————————— Note on the number of sensible proper motions to be detected after a given interval..... 394
Buss, A. A., Report of his observatory, 1922..... 271
Buxton, Arnold, Note on the effect of astigmatism on star-discs..... 475

Cambridge Observatory, Report of the proceedings of the, 1921–22..... 263
Cambridge University, Solar Physics Observatory, Report of the proceedings of the, 1921–22..... 264
Candidates proposed..... 1, 86, 112, 220, 321, 349, 381, 427
Comets:—
Council note on the discovery of, in 1922..... 287
Comet Pons-Winnecke: perturbations by Jupiter, C. J. Merfield..... 374
Cooke, F. B., W. E. Cooke and, Note on the reception of wireless time-signals 424
Cooke, W. E,, The total solar eclipse of 1922 September 21..... 511
——————————— and F. B. Cooke, Note on the reception of wireless time-signals 424
Cortie, Rev. A. L., Solar and terrestrial magnetic phenomena, 1913–21..... 204
Crommelin, A. C. D., Sir Frank Dyson and, The Greenwich observations of the moon, 1751–1922..... 359
——————————— and R. T. Cullen, Deduction of the moon's parallactic inequality and allied terms from the Greenwich lunar observations 1900 to 1922..... 458
Cullen, R. T., A. C. D. Crommelin and, Deduction of the moon's parallactic inequality and allied terms from the Greenwich lunar observations 1900 to 1922..... 458

PAGE

Davidson, C., and W. M. H. Greaves, The influence of atmospheric dispersion on photographic determinations of stellar parallax..... 56
Development of faint images by fogging, The, S. R. Pike..... 493
Dobson, G. M. B., F. A. Lindemann and, Note on the photography of meteors 163
Douglas, A. V., A. S. Eddington and, The progression of stellar velocity with absolute magnitude..... 112
Dreyer, J. L. E., Addresses delivered by, on the award of the Jackson-Gwilt Medal and Gift to Mr. A. Stanley Williams and to Mr. W. S. Franks 428
Dunsink Observatory, Report of the Proceedings of the, 1921–22..... 266
Durham University Observatory, Report of the proceedings of the, 1921–22..... 266
Dyson, Sir Frank, and A. C. D. Crommelin, The Greenwich observations of the moon, 1751–1922..... 359

Eddington, A. S., On the absorption of radiation inside a star..... 32
——— —— ——— Applications of the theory of the stellar absorption-coefficient 98
——————— Address delivered by, on the award of the Gold Medal to Professor Albert Abraham Michelson..... 309
——————— The problem of electron-capture in the stars..... 431
——— —— ——— and A. V. Douglas, The progression of stellar velocity with absolute magnitude..... 112
Edinburgh, Royal Observatory, Report of the proceedings of the, 1922..... 263
Edwards, D. L., Spectroscopic parallaxes of the hotter stars..... 47
Erratum..... 480
Espin, Rev. T. E., Micrometrical measures of double stars (17th series)..... 154
——————— Report of his observatory (Wolsingham), 1921–22..... 272

Fellows elected..... 1, 85, 111, 220, 321, 349, 381, 427
Fotheringham, J. K., The secular acceleration of the moon's mean motion as determined from occultations and conjunctions in the *Almagest* (a correction)..... 370
Fowler, R. H., and E. A. Milne, The intensities of absorption lines in stellar spectra, and the temperatures and pressures in the reversing layers of stars..... 403

Geophysical discussions and publications..... 231
Gifts to the general funds of the Society..... 226
Gold Medal of the Society awarded to Professor Albert Abraham Michelson for his applications of the interferometer to astronomical measurements 219, 309
Greaves, W. M. H., The effect of the second-order terms in differential aberration on determinations of stellar parallax..... 60
——— ——————— The problem of disturbed motion about the equilateral triangle equilibrium point and its bearing on the Goldsbrough theory of Saturn's ring..... 71
——————— C. Davidson and, The influence of atmospheric dispersion on photographic determinations of stellar parallax..... 56
Greenwich, Royal Observatory, Stellar parallaxes..... 64
——————————— Mean areas and heliographic latitudes of sun-spots in the year 1919..... 94
——————————————— Report of the proceedings of the, 1921–22..... 260
——————————————— Mean areas and heliographic latitudes of sun-spots in the year 1920..... 345
——— ——————————— Mean areas and heliographic latitudes of sun-spots in the year 1921..... 400

Helwân Observatory, Egypt, Report of the proceedings of the, 1921–22..... 277
Hertzsprung, Ejnar, Addendum to note on the motion of the Magellanic Clouds 348
Hyderabad, Nizamiah Observatory, Report of the proceedings of the, 1921–22... 278

International Astronomical Union, Council note on..... 304

PAGE

Jackson, J., The W. Struve (Σ) double stars..... 4
——————— The Otto Struve (OΣ) double stars..... 436
——————— Hypothetical parallaxes of some Aitken and Hussey double stars.... 441
——————— The masses of visual binary stars of different spectral types..... 444
Jackson-Gwilt Medals, Presentation of..... 428
Jeans, J. H., On the tidal distortion of rotating nebulæ..... 453
——————— The mechanism and structure of planetary nebulæ..... 481
Jeffreys, Harold, The constitution of the four outer planets..... 350
——————— On Jeans's theory of the origin of spiral nebulæ..... 449
Johannesburg, Union Observatory, Report of the proceedings of, 1921–22..... 281

Kodaikánal and Madras Observatories, Report of the proceedings of the, 1922... 279

Library, List of donors to the..... 307
——————— List of additions to the..... [191]
Lindblad, B., Note to Dr. Knut Lundmark's and Dr. W. J. Luyten's paper on the determination of the colour-equivalent of a star..... 97
——————— On the intensity-distribution in short grating spectra and objective-prism spectra as a function of spectral type and absolute magnitude..... 503
Lindemann, F. A., Note on the absorption of radiation inside a star..... 332
——————— Note on the constitution of the spiral nebulæ..... 354
——————— and G. M. B. Dobson, Note on the photography of meteors 163
Liverpool Observatory, Bidston, Report of the proceedings of the, 1922..... 267
Lockyer, W. J. S., Photograph of a bright meteor..... 92
——————— The spectrum of ζ Tauri..... 326
Lundmark, Knut, and W. J. Luyten, Note on the determination of absolute magnitude from λ_e and λ_m..... 470
Luyten, W. J., Knut Lundmark and, Note on the determination of absolute magnitude from λ_e and λ_m..... 470

Macklin, Rev. H. E., Note on the proper motions of stars of the clusters *h* and χ Persei..... 79
Magellanic Clouds, Addendum to note on the motion of, Ejnar Hertzsprung.... 348
Mars, Note on the occultation of, 1923 January 22, Sydney Observatory..... 379
Melbourne Observatory, Report of the proceedings of the, 1921–22..... 283
Merfield, C. J., Comet Pons-Winnecke: perturbations by Jupiter..... 374
Meteors: Orbits of the Epsilon Arietis Meteors, C. P. Olivier..... 87
——————— Photograph of a bright meteor, W. J. S. Lockyer..... 92
——————— Note on the photography of meteors, F. A. Lindemann and G. M. B. Dobson..... 163
——————— The disturbing action of the earth on meteoric showers, G. Shaju..... 341
Milne, E. A., The equilibrium of a rotating star..... 118
——————— R. H. Fowler and, The intensities of absorption lines in stellar spectra, and the temperatures and pressures in the reversing layers of stars..... 403
Minor Planets, Council note on the discovery of, in 1922..... 286
Moon, The Greenwich observations of the, 1751–1922, Sir Frank Dyson and A. C. D. Crommelin..... 359
——————— The secular acceleration of the moon's mean motion as determined from occultations and conjunctions in the *Almagest* (a correction), J. K. Fotheringham..... 370
——————— Occultations of stars by the, 1923 January 27, University Observatory, Oxford..... 373
——————— Deduction of the moon's parallactic inequality and allied terms from the Greenwich lunar observations 1900 to 1922, A. C. D. Crommelin and R. T. Cullen..... 458

Nebulæ, The galactic distribution of the spiral nebulæ, with special reference to galactic longitude, J. H. Reynolds..... 147

PAGE

Nebulæ, Note on the constitution of the spiral nebulæ, F. A. Lindemann..... 354
———— Note on Professor Lindemann's paper on the constitution of the spiral nebulæ, J. H Reynolds..... 382
———— On Jeans's theory of the origin of spiral nebulæ, Harold Jeffreys..... 449
———— On the tidal distortion of rotating nebulæ, J. H. Jeans..... 453
———— The mechanism and structure of planetary nebulæ, J. H. Jeans..... 481
Newbegin, A M., Report of his observatory, 1922..... 272

Obituary Notice :—
Wesley, William Henry (Assistant Secretary)..... 255
Obituary Notices : Associates :—
Cohn, Fritz..... 249
Kapteyn, Jacobus Cornelius..... 250
Obituary Notices : Fellows :—
Adames, Henry Bridger..... 232
Christie, Sir William H. M..... 233
Grove-Hills, Edmond Herbert..... 241
Lambert, Carlton John..... 244
Pearce, Sir Robert..... 245
Smith, Charles Michie..... 245
Stutter, Edward John..... 246
Tupman, George Lyon..... 247
Waddell, Peter Hately..... 248
Waterhouse, James..... 249
Observatories, Work at various: see reports of individual observatories (or observers) under their respective headings.
Occultation of Aldebaran, 1923 January 27, Radcliffe Observatory, Oxford..... 480
Occultations of stars by the moon, 1923 January 27, University Observatory, Oxford..... 373
Officers and Council, List of, 1923–24..... 319
Olivier, C. P., Orbits of the Epsilon Arietis Meteors..... 87
Optics : Note on the effect of astigmatism on star-discs, Arnold Buxton..... 475
Ottawa, Dominion Observatory, Report of the proceedings of the, 1922..... 274
Oxford, Radcliffe Observatory, Report of the proceedings of the, 1921–22..... 268
———————— Occultation of Aldebaran, 1923 January 27..... 480
Oxford University Observatory, Report of the proceedings of the, 1922..... 269
———————— Occultations of stars by the moon, 1923 January 27..... 373

Paisley, Coats Observatory, Report of the proceedings of the, 1922..... 269
Payne, Cecilia H., Proper motions of the stars in the neighbourhood of M 36 (N.G.C. 1960)..... 334
Phillips, Rev. T. E. R., Report of his observatory, 1921–22..... 272
Pike, S. R., The development of faint images by fogging..... 493
Planets, The constitution of the four outer, Harold Jeffreys..... 350
Prentice, Edward, R. Vulpeculæ, Ch. 7560, H. 205923. A short analysis of the observations of Knott and others..... 377
Presents announced..... 3, 86, 112, 321, 350, 382, 428
Progress and present state of the Society..... 221
Publications of the Society..... 230
———— Geophysical, and discussions..... 231

Report of the Council to the hundred and third Annual General Meeting of the Society..... 221
Reynolds, J. H., The galactic distribution of the spiral nebulæ, with special reference to galactic longitude..... 147
———————— Report of his observatory, 1921–22..... 273
———————— Note on Professor Lindemann's paper on the constitution of the spiral nebulæ..... 382
Roberts, A. W., Report of his observatory (Lovedale, S. Africa), 1922..... 281
Ross, A. D., and R. D. Thompson, On the recent light changes of Beta Ceti... 515

PAGE
Sampson, R. A., On the estimation of the continuous spectrum of stars..... 174
Saturn, The problem of disturbed motion about the equilateral triangle equilibrium point and its bearing on the Goldsbrough theory of Saturn's ring, W. M. H. Greaves..... 71
——— Note on the occultation of, 1923 May 26, Sydney Observatory..... 516
Second-order terms, The effect of the, in differential aberration on determinations of stellar parallax, W. M. H. Greaves..... 60
Shajn, G., The systematic motions of stars of type A deduced from their radial velocities..... 338
——— The disturbing action of the earth on meteoric showers..... 341
Sidmouth, Norman Lockyer Observatory, Report of the proceedings of the, 1921–22 267
Smart, W. M., The construction of a chart giving the angular distances of stars from, and the position-angles relative to, the ant-apex of the solar motion..... 465
——— The proper motion of a faint star near η Cassiopeiæ..... 501
South Kensington, Imperial College, Report of the proceedings of the, 1921–22 270
Star Clusters: Note on the proper motions of stars of the clusters h and χ Persei, Rev. H. E. Macklin..... 79
Stars, Constitution and Evolution of: On the absorption of radiation inside a star, A. S. Eddington..... 32
——— Applications of the theory of the stellar absorption coefficient, A. S. Eddington..... 98
——— The equilibrium of a rotating star, E. A. Milne..... 118
——— Note on the absorption of radiation inside a star, F. A. Lindemann..... 332
——— The intensities of absorption lines in stellar spectra, and the temperatures and pressures in the reversing layers of stars, R. H. Fowler and E. A. Milne..... 403
——— The problem of electron-capture in the stars, A. S. Eddington..... 431
Stars, Distribution and Motions of: The progression of stellar velocity with absolute magnitude, A. S. Eddington and A. V. Douglas..... 112
——— The systematic motions of stars of type A deduced from their radial velocities, G. Shajn..... 338
——— The construction of a chart giving the angular distances of stars from, and the position-angles relative to, the ant-apex of the solar motion, W. M. Smart..... 465
——— Stellar velocity and absolute magnitude: note on a paper by Professor Eddington and Miss Douglas, W. S. Adams and G. Strömberg..... 474
Stars, Double and Multiple: The W. Struve (Σ) double stars, J. Jackson..... 4
——— Micrometrical measures of double stars (17th series), Rev. T. E. Espin..... 154
——— Council note on..... 300
——— The Otto Struve ($O\Sigma$) double stars, J. Jackson... 436
——— Hypothetical parallaxes of some Aitken and Hussey double stars, J. Jackson..... 441
——— The masses of visual binary stars of different spectral types, J. Jackson..... 444
Stars, New: Observations of Novæ, 1922, W. H. Steavenson..... 160
——— Observations of Nova Persei (1901) in 1922-23, W. H. Steavenson 397
Stars, Parallaxes of: Spectroscopic parallaxes of the hotter stars, D. L. Edwards 47
——— The influence of atmospheric dispersion on photographic determinations of stellar parallax, C. Davidson and W. M. H. Greaves 56
——— The effect of the second-order terms in differential aberration on determinations of stellar parallax, W. M. H. Greaves..... 60
——— Stellar parallaxes determined at the Royal Observatory, Greenwich..... 64
Stars, Proper Motions of: Fourteenth note on the number of faint stars with large proper motions on plates of the Oxford Astrographic Catalogue for zones +25°, +26°, and +27°, F. A. Bellamy..... 67

PAGE

Stars, Proper Motions of: Note on the proper motions of stars of the clusters h and χ Persei, Rev. H. E. Macklin 79
——— Proper motions of the stars in the neighbourhood of M 36 (N.G.C. 1960), Cecilia H. Payne 334
——— Note on the number of sensible proper motions to be detected after a given interval, F. A. Bellamy 394
——— The proper motion of a faint star near η Cassiopeiæ, W. M. Smart 501
Stars, Spectra of: Note to Dr. Knut Lundmark's and Dr. W. J. Luyten's paper on the determination of the colour-equivalent of a star, B. Lindblad 97
——— Lines of unknown origin in various celestial spectra, F. E. Baxandall 166
——— On the estimation of the continuous spectrum of stars, R. A. Sampson 174
——— Council note on stellar spectroscopy in 1922 294
——— Wave-lengths of silicon lines occurring in the spectra of B-type stars, H. Barrell 322
——— The spectrum of ζ Tauri, W. J. S. Lockyer 326
——— The intensities of absorption lines in stellar spectra, and the temperatures and pressures in the reversing layers of stars, R. H. Fowler and E. A. Milne 403
——— Note on the determination of absolute magnitude from λ_e and λ_m, Knut Lundmark and W. J. Luyten 470
——— On the intensity-distribution in short grating spectra and objective-prism spectra as a function of spectral type and absolute magnitude, B. Lindblad 503
Stars, Variable: R. Vulpeculæ, Ch. 7560, H. 205923. A short analysis of the observations of Knott and others, Edward Prentice 377
——— On the recent light changes of Beta Ceti, A. D. Ross and R. D. Thompson 515
Steavenson, W. H., Observations of Novæ, 1922 160
——— Report of his observatory, 1922 273
——— Observations of Nova Persei (1901) in 1922–1923 397
Stonyhurst College Observatory, Report of the proceedings of the, 1922 270
Strömberg, G., W. S. Adams and, Stellar velocity and absolute magnitude: note on a paper by Professor Eddington and Miss Douglas 474
Sun, Eclipses of the: The proximity of Venus to the sun during the total solar eclipse of 1923 September 10, J. H. Worthington 424
——— The total solar eclipse of 1922 September 21, W. E. Cooke 511
Sun, Spectrum of, etc.: Council note on solar research in 1922 291
Sun-spots and Faculæ: Mean areas and heliographic latitudes of sun-spots in the year 1919, Royal Observatory, Greenwich 94
——— Solar and terrestrial magnetic phenomena, 1913–21, Rev. A. L. Cortie 204
——— Comparison of sun-spot areas and terrestrial magnetic horizontal force-ranges, 1911–21, Rev. B. G. Swindells 215
——— Council note on solar activity in 1922 290
——— Mean areas and heliographic latitudes of sun-spots in the year 1920, Royal Observatory, Greenwich 345
——— Mean areas and heliographic latitudes of sun-spots in the year 1921, Royal Observatory, Greenwich 400
Swindells, Rev. B. G., Comparison of sun-spot areas and terrestrial magnetic horizontal force-ranges, 1911–21 215
Sydney Observatory, Report of the proceedings of the, 1922 284
——— Note on the occultation of Mars, 1923 January 22 379
——— Note on the occultation of Saturn, 1923 May 26 516

Thompson, R. D., A. D. Ross and, On the recent light changes of Beta Ceti ... 515
Treasurer's account for 1922 222
Trust funds 227

PAGE

Turner, H. H., The stellar magnitude scales of the Astrographic Catalogue. Seventeenth note: the San Fernando magnitudes for $-3°$ and $-5°$, and the Paris magnitudes for $+22°$, $+23°$, and $+24°$..... 385

Venus, The proximity of, to the sun during the total solar eclipse of 1923 September 10, J. H. Worthington..... 424
Victoria, B.C., Dominion Astrophysical Observatory, Report of the proceedings of the, 1921–22..... 275

Wireless time-signals, Note on the reception of, W. E. Cooke and F. B. Cooke 424
Worthington, J. H., The proximity of Venus to the sun during the total solar eclipse of 1923 September 10..... 424

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CONTENTS.

	PAGE
Fellows elected	1
Candidates proposed	*ib.*
Presents announced	3
J. Jackson, The W. Struve (Σ) Double Stars	4
A. S. Eddington, On the Absorption of Radiation inside a Star	32
D. L. Edwards, Spectroscopic Parallaxes of the Hotter Stars	47
C. Davidson and **W. M. H. Greaves**, The Influence of Atmospheric Dispersion on Photographic Determinations of Stellar Parallax	56
W. M. H. Greaves, The Effect of the Second-Order Terms in Differential Aberration on Determinations of Stellar Parallax	60
Royal Observatory, Greenwich, Stellar Parallaxes	64
F. A. Bellamy, Fourteenth Note on the Number of Faint Stars with Large Proper Motions on Plates of the Oxford Astrographic Catalogue for Zones $+25°$, $+26°$, and $+27°$	67
W. M. H. Greaves, The Problem of Disturbed Motion about the Equilateral Triangle Equilibrium Point and its Bearing on the Goldsbrough Theory of Saturn's Ring	71
Rev. H. E. Macklin, Note on the Proper Motions of Stars of the Clusters h and χ Persei	79
E. M. Antoniadi, Note on a Paper by M. Shajn	83

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